Mobile station localization by combining measurements from different sources including reliability information

Carsten Fritsche*, Anja Klein* and Heiko Schmitz**
*Technische Universität Darmstadt, Fachgebiet Kommunikationstechnik, Merckstr. 25, 64283 Darmstadt, Germany
**Siemens AG, COM MN PG NT AM 3, Siemensdamm 62, 13627 Berlin, Germany
E-Mail: c.fritsche@nt.tu-darmstadt.de, a.klein@nt.tu-darmstadt.de, heiko.schmitz@siemens.com

Abstract

In recent years, the demand for localization methods (LMs) offering precise mobile station (MS) position estimates has increased due to the introduction of location based services and the requirement of promptly alerting rescue services in case of emergency. In dense urban and indoor scenarios, the existing Global Navigation Satellite System (GNSS) based LMs fail and the cellular radio network (CRN) based LMs do not reach the desired localization accuracy. Each LM is based on the combination of information on the MS position provided by measured values (MVs). In this paper, a novel hybrid LM is proposed that is based on the combination of information on the MS position available from different MVs of the CRN and GNSS. The problem of combining information on the MS position available from different MVs is solved by determining for each MV the conditional probability density function (cpdf) of the MS position conditioned on the given MV. The cpdf for each MV can be determined from a model for the MV (MM) that is introduced. The MM contains the functional relationship between the MV and the MS position, and a model for the errors each MV is affected with. The most probable MS position can be found by combining the cpdfs of the different MVs. As an example, the MMs and cpdfs of MVs from the CRN, namely Received Signal Strength and Round Trip Time and from the GNSS, namely Time of Arrival are presented. Based on a GSM and GPS network scenario, the proposed hybrid LM is verified by means of simulations. The simulation results show that combining MS position information of MVs from the CRN and GNSS significantly enhances the localization accuracy compared to combining only the MS position information of MVs from the CRN.

1 Introduction

The localization of mobile stations (MS) has become important for many applications in the last few years. The applications range from the provision of precise MS location estimates, required in case of emergency calls [1], to commercially available location based services as, e.g., fleet management, fraud detection, friend finder, navigation and location based information advertising.

The existing localization methods (LMs), e.g., Global Navigation Satellite System (GNSS) based and cellular radio network (CRN) based LMs, rely on combining the information on the MS position provided by measured values (MV). Having MVs from multiple base stations (BSs) or satellites (SATS) available, the MS position can be determined [2], [3]. The GNSS based LMs are based on measuring the Time of Arrival (TOA), e.g., in the Global Positioning System (GPS) [2]. A three dimensional (3-D) MS position can be determined if TOA MVs from at least four SATs are available [2]. The CRN based LMs are based on measuring, e.g., the Received Signal Strength (RSS), Round Trip Time (RTT) or Angle of Arrival (AOA) [3]. The MS position can be determined for each LM if MVs from at least three BSs are available [3]. There exist also LMs that combine different MVs from the CRN [4], [5].

In dense urban and indoor scenarios, the number of SATs in view is often insufficient to determine a 3-D or even a two dimensional (2-D) MS position due to the attenuation or complete shadowing of the SAT signals [2]. The CRN based LMs are available almost everywhere, but do not reach the accuracy of the GNSS based LMs. In dense urban and indoor scenarios, a promising approach is to combine the information provided by the MVs from the CRN and the GNSS, in order to achieve a precise MS position estimate [6], [7]. The resulting LM is termed hybrid LM. Additionally, taking into account information provided by different MVs from the CRN can further enhance the accuracy and availability of MS position estimates.

In this paper, a novel hybrid LM for dense urban and indoor scenarios is proposed that is based on combining MS position information available from MVs of the CRN with MS position information available from the MVs of the GNSS. The problem of combining the MS position information available from different MVs is solved by determining for each MV the conditional probability density function (cpdf) of the MS position conditioned on the given MV over a 2-D or 3-D space.
The cpdf of each MV can be determined from a model for the MV (MM) relating the MS position with the MV. The MM introduced in this paper contains the functional relationship between the MV and the MS position and a model for the errors each MV is affected with. Then it is shown how, in general, the cpdfs can be combined, from which the MS position can be then estimated. As an example, the MM and the errors of the RSS, RTT and TOA MVs are presented based on the MM of MVs that can be obtained from the Global System of Mobile Communication (GSM) and GPS. The proposed hybrid LM using the MVs RSS, RTT and TOA is then tested in an urban scenario with fixed BS and SATs positions by means of simulations. The advantage of the proposed hybrid LM over the LM that only uses the MVs TA and RSS is demonstrated.

The paper is organized as follows: In section 2, the MM is introduced from which the cpdf of the MS position conditioned on the MV is derived. Then, it is shown how, in general, cpdfs can be combined. In section 3, the MM and the cpdfs of the RSS, RTT, and TOA MVs are determined. In section 4, the simulation scenario is presented. In section 5, the simulation results of combining cpdfs of different MV are presented. Section 6 concludes the paper.

2 Combination of conditional probability density functions of different measured values

2.1 Conditional probability density functions

In the following, the MM relating the MV with the MS position is introduced. Based on the MM, the cpdf of the MS position conditioned on a given MV is determined.

Below, an observation domain \( G \subset \mathbb{R}^3 \) in the 3-dimensional space is defined. The observation domain \( G \) is assumed to contain \( K_{BS} \) fixed BS positions \( \bar{x}_{BS}^{i}, i = 1, \ldots, K_{BS} \), \( K_{SAT} \) fixed SAT positions \( \bar{x}_{SAT}^{i}, i = 1, \ldots, K_{SAT} \), and the MS position \( \bar{x}_{MS} \in G \) which has to be estimated.

It is further assumed that \( N \) MVs, \( n_i \), \( i = 1, \ldots, N \), are available, each depending on the MS position \( \bar{x}_{MS} \). The MV \( m_i \) is affected by errors that are assumed to be additive. The errors can be described statistically by a random variable \( n_i \) with the probability density function (pdf) \( p_{n_i}(n_i) \). In the following, the functional relationship between the MV \( m_i \) and the MS position \( \bar{x}_{MS} \), given by \( m_i = f_i(\bar{x}_{MS}, n_i) \) is termed characteristic function (CF) [9]. The CF \( f_i(\bar{x}_{MS}, n_i) \) is composed of the error-free CF \( f_i(\bar{x}_{MS}) \) relating the error-free MV \( \bar{m}_i \) to the MS position \( \bar{x}_{MS} \), i.e., \( \bar{m}_i = f_i(\bar{x}_{MS}) \), and a function that statistically models the errors \( n_i \) each MV is affected with. Thus, the CF can be described by:

\[
m_i = f_i(\bar{x}_{MS}) + n_i.
\]

(1)

The MVs considered in this paper are functionally related to the distance \( d_{i,MS}^{(k)} \), \( k \in \{BS, SAT\} \), between the MS position \( \bar{x}_{MS} \) and the BS position \( \bar{x}_{BS}^{(k)} \) or SAT position \( \bar{x}_{SAT}^{(k)} \). Thus, in the previous considerations the distance \( d_{i,MS}^{(k)} \) can be used instead of the MS position \( \bar{x}_{MS} \). The distance \( d_{i,MS}^{(k)} \) is given by:

\[
d_{i,MS}^{(k)} = \|\bar{x}_{MS}^{(k)} - \bar{x}_{MS}\|_2 \in \mathbb{R}^+. \tag{2}
\]

For simplicity, the expression \( d_{i,MS}^{(k)} \) is reduced to \( d_{i,MS} \) in the following. The error-free CF \( g_i(d_{i,MS}) \) relates the error-free MV \( \bar{m}_i \) to the distance \( d_{i,MS} \), i.e., \( \bar{m}_i = g_i(d_{i,MS}) \). Eq. (1) can be thus rewritten as:

\[
m_i = g_i(d_{i,MS}) + n_i. \tag{3}
\]

From the available MVs \( m_i \), the corresponding cpdf \( p(d_{i,MS} = d_{i,0} | m_i) \) for all possible MS distances \( d_{i,0} \) can be found. The possible MS distance \( d_{i,0} \) is the Euclidean distance between the BS or SAT position \( \bar{x}_{MS}^{(k)} \) and the possible MS position \( \bar{x}_0 \in G \) analogous to Eq. (2). In order to determine the cpdf \( p(d_{i,MS} = d_{i,0} | m_i) \) from Eq. (3), the distribution \( p(d_{i,MS} = d_{i,0}) \) of the possible distances \( d_{i,0} \) in \( \mathbb{R}^+ \) has to be known, or an assumption has to be made. For simplicity, the expression \( d_{i,MS} = d_{i,0} \) is replaced by \( d_i \) in the following. Assuming that \( n_i \) and \( g_i(d_i) \) are statistically independent,

\[
p(m_i | g_i(d_i)) = p_{n_i}(m_i - g_i(d_i)) \tag{4}
\]

results from Eq. (3). The cpdf \( p(g_i(d_i) | m_i) \) can be found from Eq. (4) by applying Bayes’ rule [10]. Assuming that \( g_i(d_i) \) is strictly increasing, the cpdf \( p(d_i | m_i) \) can be found from \( p(g_i(d_i) | m_i) \) by transformation [10]:

\[
p(d_i | m_i) = \frac{1}{\int_{0}^{\infty} p(d_i) | m_i | g_i(d_i)) \cdot g_i(d_i) \dd d_i} \cdot g_i(d_i). \tag{5}
\]

In a next step, the cpdf \( p(\bar{x}_{MS} = \bar{x}_0 | m_i) \) of the possible MS positions \( \bar{x}_0 \) over a 3-D space has to be determined. This can be done, e.g., by determining the cpdf \( p(\bar{x}_{MS} = \bar{x}_0 | m_i) \) in dependence of the random variables \( d_{i,0}, \phi_{i,0} \) and \( \theta_{i,0} \) that can be considered as spherical coordinates. The BSs or SATs are located in the centre of the coordinate system. The cpdf of \( d_{i,0} \) is given by Eq. (5). The cpdfs of the elevation \( \phi_{i,0} \) and azimuth \( \theta_{i,0} \) are given by \( p(\phi_{i,MS} = \phi_{i,0} | m_i) \) and \( p(\theta_{i,MS} = \theta_{i,0} | m_i) \), to have to be known or an assumption has to be made. For simplicity, the expressions \( \theta_{i,MS} = \theta_{i,0} \) and \( d_{i,MS} = d_{i,0} \) are replaced by \( \theta_i \) and \( \phi_i \). Assuming that \( d_i, \theta_i, \) and \( \phi_i \) are statistically independent,

\[
p(d_i, \theta_i, \phi_i | m_i) = p(d_i | m_i) \cdot p(\theta_i | m_i) \cdot p(\phi_i | m_i) \tag{6}
\]
results. In order to combine the cpdfs of the different MVs, the cpdfs have to be determined for a common coordinate system.

2.2 Combination of conditional probability density functions

In the following it is shown how, in general, the cpdf of the MS position $\hat{x}_{MS}$ conditioned on all available MVs can be determined. Having $N$ different MVs $m_i$, $i = 1, \ldots, N$, the cpdf of the MS position $\hat{x}_{MS}$ conditioned on all available MVs $m_1, \ldots, m_N$, given by $p(\hat{x}_{MS} = \hat{x}_0 | m_1, \ldots, m_N)$, can be found from the cpdfs $p(\hat{x}_{MS} = \hat{x}_0 | m_j)$ of the MVs $m_j$. For simplicity, the expression $\hat{x}_{MS} = \hat{x}_0$ is replaced by $\hat{x}$ in the following. With the joint pdf $p(\hat{x}, m_1, \ldots, m_N)$ and the a-priori pdf $p(m_1, \ldots, m_N)$,

$$p(\hat{x} | m_1, \ldots, m_N) = \frac{p(\hat{x}, m_1, \ldots, m_N)}{p(m_1, \ldots, m_N)} \quad (7)$$

holds [10]. The joint pdf $p(\hat{x}, m_1, \ldots, m_N)$ can be further decomposed into

$$p(\hat{x}, m_1, \ldots, m_N) = p(\hat{x}) \prod_{i=1}^{N} p(m_i | \hat{x}, m_{i+1}, \ldots, m_N) \quad (8)$$

with the a-priori pdf $p(\hat{x})$ [10]. In the following, it is assumed that for a given MS position $\hat{x}_{MS}$, the different MVs $m_j$ and $m_i$, $i = 1, \ldots, N$, $j = 1, \ldots, N$, $i \neq j$, are statistically independent of each other. This assumption is considered to be a good approximation if the MVs result from different LMs that suffer from different errors. As a consequence, each of the cpdfs $p(m_i | \hat{x}, m_{i+1}, \ldots, m_N)$, $i = 1, \ldots, N$, is independent of the conditions $m_{i+1}, \ldots, m_N$ and, thus, can be replaced by

$$p(m_i | \hat{x}, m_{i+1}, \ldots, m_N) = p(m_i | \hat{x}) \quad \text{for} \quad i = 1, \ldots, N \quad (9)$$

Consequently, the cpdf $p(\hat{x} | m_1, \ldots, m_N)$ can be expressed as a function of the cpdfs $p(\hat{x} | m_j)$ of the MVs $m_j$. By applying Bayes’ rule and taking into account that the integral of $p(\hat{x} | m_1, \ldots, m_N)$ over $G$ equals one,

$$p(\hat{x} | m_1, \ldots, m_N) = \int_G \frac{p(\hat{x}) \prod_{i=1}^{N} p(m_i | \hat{x})}{\int_G p(\hat{x}) \prod_{i=1}^{N} p(m_i | \hat{x})} \, d\hat{x} \quad (10)$$

results. The equation above equals the expression that can be found in [8] if $p(\hat{x})$ is assumed to be constant. The integral given in the denominator of Eq. (10) yields a constant factor. Assuming further that no a-priori information about the MS position is available, i.e. $p(\hat{x})$ is assumed to be uniformly distributed in $G$, Eq. (10) reduces to

$$p(\hat{x} | m_1, \ldots, m_N) = C \prod_{j=1}^{N} p(\hat{x} | m_j), \quad (11)$$

with $C$ as a constant factor. In order to find the most probable MS position $\hat{x}$, the cpdf $p(\hat{x} | m_1, \ldots, m_N)$ has to be maximized, leading to

$$\hat{x} = \arg \max_{\hat{x}} p(\hat{x} | m_1, \ldots, m_N). \quad (12)$$

3 Examples of conditional probability density functions

3.1 Introduction

The theoretical considerations of section 2 are applied to MVs that can be obtained from the CRN, namely RSS and RTT, and from the GNSS, namely TOA. For each MV, the CF is determined. Based on MVs that can be obtained from the GSM and GPS network, the corresponding cpdfs are presented.

3.2 Received Signal Strength

In CRNs, the RSS value provides a mean value of the strength of the signal received by the MS from the BS [11]. If the BS transmit power is known, the attenuation of the signal, also known as path loss, can be determined. In the following, the CF of the mean path loss MV $m_{pl}$ is determined. The error-free CF $g_{pl}(d_{BS,MS})$ in dB relating the error-free MV $\tilde{m}_{pl}$ to the distance $d_{BS,MS}$ is given by

$$\tilde{m}_{pl} = g_{pl}(d_{BS,MS}) = 10 \log_{10} \left( \frac{1}{\beta} \left( \frac{d_{BS,MS}}{\alpha} \right)^a \right) \quad (13)$$

[5], where $\alpha$ and $\beta$ are model coefficients. The model coefficients depend on the propagation conditions and the transmitter’s antenna settings and can be either found from empirical measurements or from the Hata-Okumura path loss formula [12].

The MV $m_{pl}$ is affected by errors due to fast fading and slow fading. Because the MV $m_{pl}$ is averaged over several measurements, the fast fading is averaged out. Thus, it is assumed that the MV $m_{pl}$ is only affected by errors due to slow fading. The error in dB due to slow fading can be modelled as an additive zero-mean Gaussian random variable $n_{pl}$ with standard deviation $\sigma_{SF}$ [11]. The CF of the MV $m_{pl}$ in dB is thus given by

$$m_{pl} = g_{pl}(d_{BS,MS}) + n_{pl} \quad (14)$$

In order to determine $p(d_{BS,MS} = d_{BS,0} | m_{pl})$ from Eq. (14), the a-priori pdf $p(d_{BS,MS} = d_{BS,0})$ of the possible distance $d_{BS,0}$ has to be known. Although it is assumed that $p(\hat{x})$ is uniformly distributed in order
to obtain Eq. (11), the possible distance $d_{BS,0}$ is assumed to be uniformly distributed to simplify calculations. In the following, the expression $d_{BS,MS} = d_{BS,0}$ is replaced by $d_{BS}$ for simplicity. The corresponding cpdf $p(d_{BS} \mid m_{pl})$ is lognormal [10] with the following parameters:

$$
\mu_{pl} = \frac{m_{pl} \cdot \log 10}{10 \alpha} + \log \beta + \sigma_{pl}^2, \quad \sigma_{pl} = \sqrt{\frac{\log 10}{10 \alpha}} \cdot (15)
$$

As an example, the cpdf $p(d_{BS} \mid m_{pl})$ for different MVs $m_{pl}$ based on the Hata-Okumura model [12] is depicted in Fig. 1.

As can be seen from Fig. 1, the graph of the cpdf becomes broader as the path loss increases, i.e., the reliability of the information about the MS distance $d_{BS}$ to the BS decreases as the path loss increases.

### 3.3 Round Trip Time

The round trip time (RTT) is a parameter that is used in radio networks to synchronize the transmitted bursts of MSs to the frame at the receiving BS [11]. The synchronization is accomplished by measuring the RTT at the BS which is then rounded to the nearest integer bit period $T_b$. In the following, the CF for the RTT MV that is available from the GSM network which is also known as Timing Advance (TA) MV $m_{TA}$ is determined. The error-free CF $g_{TA}(d_{BS,MS})$ relating the error-free MV $\tilde{m}_{TA}$ to the distance $d_{BS,MS}$ is given by

$$
\tilde{m}_{TA} = g_{TA}(d_{BS,MS}) = \frac{d_{BS,MS}}{c_0}, \quad (16)
$$

where $c_0 = 3 \cdot 10^8$ m/s is the speed of light. Because the TA is given as integer multiple of the bit period $T_b = 3.69 \mu s$, the distance $d_{BS,MS}$ can be only represented with a finite resolution $\Delta = 553.5 \mu m$.

Each MV $m_{TA}$ is affected by errors resulting from the propagation conditions - line-of-sight (LOS) or non-line-of-sight (NLOS) situation - and inaccuracies of the measurement equipment. It is assumed that the errors can be modelled as a Gaussian random variable $n_{TA}$ with mean $\mu_{TA}$ accounting for the error due to NLOS situations and standard deviation $\sigma_{TA}$. The CF of the MV $m_{TA}$ is thus given by

$$
m_{TA} = g_{TA}(d_{BS,MS}) + n_{TA},
$$

Eq. (17) represents the MV $m_{TA}$ normalized to the bit period $T_b$ without considering the resolution constraints. In order to take into account the resolution constraints, the conditional probability $P(m_{TA} \mid g_{TA}(d_{BS}))$ that the MV $m_{TA}$ is inside the tolerance of a bit interval $T_b$ is determined from Eq. (17). From $P(m_{TA} \mid g_{TA}(d_{BS}))$ and assuming that the distance $d_{BS,0}$ is uniformly distributed, the cpdf $p(d_{BS} \mid m_{TA})$ can be found which is proportional to the following expression:

$$
p(d_{BS} \mid m_{TA}) = \frac{1}{2} \Phi(\frac{h_{+}}{\sigma_{TA}}) - \Phi(\frac{h_{-}}{\sigma_{TA}}) \quad \text{for } d_{BS} \geq 0
$$

with the following function

$$
h_z = \left( m_{TA} + \frac{z}{2} \right) \frac{T_b}{c_0} - \frac{2 \cdot d_{BS,MS}}{c_0} - \mu_{TA}, \quad z \in \{ \pm 1 \},
$$

where $\Phi(x)$ denotes the cumulative distribution function of $x$. As an example, the cpdf $p(d_{BS} \mid m_{TA})$ for different MVs $m_{TA}$ is depicted in Fig. 2.

### 3.4 Time of Arrival

The GNSS based LMs are based on measuring the TOA, e.g., in the GPS network [2]. In the following, the CF for the TOA MV $m_{GPS}$ is determined. The error-free CF $g_{GPS}(d_{SAT,MS})$ relating the error-free MV $\tilde{m}_{GPS}$ to the distance $d_{SAT,MS}$ is given by

$$
\tilde{m}_{GPS} = g_{GPS}(d_{SAT,MS}) = \frac{d_{SAT,MS}}{c_0},
$$

The MV $m_{GPS}$ is affected by errors due to delays as the signal propagates through the atmosphere, as well as resolution, receiver noise and receiver clock offsets at the MS that may delay or advance the signal [2]. Assuming that the MS is synchronized to GPS time...

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according to the available radio resource protocol [13], the errors due to the receiver clock offset are not taken into account in the CF. The distribution of the remaining errors, given by \( n_{GPS} \) is assumed to be zero-mean Gaussian with the standard deviation \( \sigma_{GPS} \) that can be approximated by the user equivalent range error [2]. Although the assumptions of synchronization and perfect knowledge of \( \sigma_{GPS} \) in the position estimation process is optimistic, it is sufficient for first simulations to show the benefit of the hybrid LM. The CF of the MV \( m_{GPS} \) is thus given by

\[
m_{GPS} = g_{GPS}(d_{SAT,MS}) + n_{GPS}.
\]

Assuming that the distance \( d_{SAT,0} \) is uniformly distributed, the cpdf \( p(d_{SAT,MS} = d_{SAT,0} \mid m_{GPS}) \) can be determined from Eq. (21). For \( d_{SAT} \geq 0 \), the cpdf \( p(d_{SAT,MS} = d_{SAT,0} \mid m_{GPS}) \) is proportional to a normal distribution with the following parameters:

\[
\mu_{SAT} = c_0 \cdot m_{GPS}, \quad \sigma_{SAT} = c_0 \cdot \sigma_{GPS}.
\]

### 3.5 Three dimensional conditional probability density function

The cpdf \( p(\vec{x} \mid m_l) \) of each MV \( m_l \), introduced in section 3.2 to 3.4, can be found from the corresponding cpdf \( p(d_i \mid m_l) \) and the distribution of the elevation \( \theta_i,0 \) and azimuth \( \varphi_i,0 \). Assuming that for all MVs, \( \theta_i,0 \) and \( \varphi_i,0 \) are both uniformly distributed and statistically independent of the distance \( d_i,0 \), the cpdf \( p(d_i, \theta_i, \varphi_i \mid m_l) \) can be found according to Eq. (6).

### 4 Simulation Scenario

The simulation scenario is composed of a 3-D observation domain \( G \) that contains \( K_{BS} = 3 \) fixed base stations, \( K_{SAT} = 2 \) fixed SATs and the MS as shown in Fig. 3. The CRN composed of the BSs and the MS is assumed to be 2-D, i.e. the BS positions as well as the MS position lie in the \( x-y \) plane, denoted by \( G_{2D} \subseteq G \). The adjacent BS positions are taken from a regular hexagonal cell grid, where omni-directional antennas of the BSs are located in the centre of the hexagons. The MS position \( \vec{x}_{MS} \) is uniformly distributed within the isosceles triangle formed by the three adjacent BSs. The isosceles triangle can be found from any combination of three adjacent BSs in a hexagonal cell grid and, thus, its area is representative for evaluating the performance of the proposed hybrid LM. The serving BS (SBS) is the BS with the smallest distance to the MS.

In urban scenarios, the MS is surrounded by multi-story buildings that prevent the MS to have three or more SATs in view. In these situations, only SATs at high elevation angles are visible to the MS. Thus, for the simulations, the two SATs with the highest elevation angles are chosen.

The fixed SAT positions are chosen from a real SAT constellation that is given for a fixed MS position at a specific date and time (Berlin, 01.08.06, 2 a.m.), as it can be provided from GPS Almanac Data. In the simulations, the MVs are generated from Eqs. (14), (17) and (21). The cpdfs and the MV are generated with the parameters of Table 1. From this it follows that Eq. (13) is known at the receiver which is not true in reality. It is assumed further that the MV and cpdf are not affected by errors due to multipath.

<table>
<thead>
<tr>
<th>Parameter Value</th>
<th>Parameter Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( v_{SAT}^{(1)} ) in ( 10^4 ) km</td>
<td>( [-2.7,-4.7,19.7] )</td>
</tr>
<tr>
<td>( v_{SAT}^{(2)} ) in ( 10^4 ) km</td>
<td>( [1.2,5.7,16.6] )</td>
</tr>
<tr>
<td>( \sigma_{vel} ) in ( km )</td>
<td>( [1.4] )</td>
</tr>
<tr>
<td>( \mu_{vel} ) in ( km )</td>
<td>( [2.4] )</td>
</tr>
<tr>
<td>( \sigma_{vel} ) in ( km )</td>
<td>( [1.5,3] )</td>
</tr>
<tr>
<td>( \beta )</td>
<td>( 2.3 \times 10^{-11} )</td>
</tr>
</tbody>
</table>

Table 1: Simulation parameters

As long as the MS position is given in 2-D, the corresponding cpdfs have to be determined for the 2-D case. This can be done by mapping the 3-D cpdf introduced in section 3.5 into the \( x-y \) plane by first expressing \( d_i,0 \cdot \theta_i,0 \) and \( \varphi_i,0 \) as functions of Cartesian coordinates and then setting \( z = 0 \). The resulting 2-D cpdfs of the different MVs can then be combined according to Eq. (11).

### 5 Simulation Results

The following combinations of MVs are investigated:

- TA from serving BS and 2 RSS from neighbouring BSs (1 TA + 2 RSS)
- TA from SBS, 2 RSS from neighbouring BSs and TOA from SAT with highest elevation angle (1 TA + 2 RSS + 1 SAT)
- TA from SBS, 2 RSS from neighbouring BSs and TOA from 2 SATs (1 TA + 2 RSS + 2 SAT)

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The most probable MS position $\hat{x}$ can be found from the cpdf $p(\hat{x}|m_1,\ldots,m_N)$ by applying the following method. First, a domain $\tilde{G} \subseteq \mathbb{Z}^N$ is defined so that it can be assured that the MS is located in $\tilde{G}$. The domain $\tilde{G}$ is then discretized to a grid with a finite number of discrete points. In a next step, the cpdf $p(\hat{x}|m_1,\ldots,m_N)$ is evaluated at the discrete points. The point that maximizes $p(\hat{x}|m_1,\ldots,m_N)$ is assigned to the most probable MS position $\hat{x}$.

In order to compare the localization accuracy of the proposed combinations of MVs with each other, the localization error is introduced as

$$\Delta = \|\hat{x} - \hat{x}_{MS}\|_2$$

where $\Delta$ is the localization error, $\hat{x}$ is the estimated MS position, and $\hat{x}_{MS}$ is the true MS position.

As both the MS position $\hat{x}$ and the true MS position $\hat{x}_{MS}$ are random variables, also the localization error $\Delta$ is a random variable. The localization accuracy is evaluated in a statistical sense by means of the complementary cumulative distribution function (CCDF) $P(\Delta > \Delta_T)$ of the localization error $\Delta$. The CCDF $P(\Delta > \Delta_T)$ describes the probability that the localization error $\Delta$ exceeds a certain threshold $\Delta_T$. The CCDFs for the different combinations of MVs are given in Fig. 4.

![Fig. 4](image_url)  
**Fig. 4** CCDF $P(\Delta > \Delta_T)$ of the localization error $\Delta$ for different combinations of MVs.  

As can be seen from Fig. 4, the LM based on TA and RSS MVs has the lowest localization accuracy. The localization accuracy can be improved by taking into account MVs from SATs. Whereas a single SAT MV slightly improves the localization accuracy compared to the combination of TA and RSS MVs, the incorporation of two SAT MVs can significantly improve localization performance. The performance of the LM based can be further improved by taking into account, e.g., sectorized BS antennas, the RSS from the SBS and other neighboring BSs, and considering more sophisticated parameters for the cpdfs of the MVs.

### 6 Conclusion

In this paper, the combination of MS position information of MVs from the CRN with MVs from the GNSS based on cpdfs has been introduced. Based on a model for the MV, the cpdfs for each MV have been determined. By simulations it could be shown that the combination of cpdfs of MVs from the CRN and GNSS can significantly improve the localization accuracy.

### 7 Acknowledgement

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### 8 References


