COOPERATIVE MIMO RELAYING WITH DISTRIBUTED SPACE-TIME BLOCK CODES

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Abstract

In this paper, space-time block codes (STBCs), which gain from spatial transmit diversity, are applied in a distributed fashion at several cooperating relay stations (RSs) with multiple transmit antennas. It is well known that non-distributed STBCs exhibit a degraded bit error rate (BER) performance in spatially correlated MIMO channels. Applying distributed STBCs in cooperative relay networks reduces the probability of correlated channel coefficients as the RSs are spatially separated. In this paper, the Chernoff bound of the BER in Rayleigh fading channels is extended to the case of correlated channel coefficients at the same relay station and different receive powers from different cooperating RSs. It is shown that the BER performance has a higher sensitivity to spatial correlation in MIMO channels than to different receive powers at the receiver from several cooperating RSs for distributed space-time coding. The theoretical results are confirmed by means of simulations.

I. Introduction

Recently, multihop and relay networks have gained a lot of attention as they provide promising solutions to the high data rate coverage requirements that appear for beyond 3G mobile radio systems [1][2]. Relay networks reduce the range problem appearing for high data rate requirements combined with high carrier frequencies, e.g., around 5GHz. In relay networks, the basic idea is to introduce a relay station (RS) which forwards data from a source node (SN) to a receive node (RN) which is out of reach of the SN. There are two prominent concepts for the transmit signal of the RS [3]. Firstly, amplify-and-forward (AF) is a low effort concept where the receive signal is stored, amplified and retransmitted by the RS. Secondly, decode-and-forward (DF) is a concept which requires a higher effort as the receive signal is decoded, re-encoded and retransmitted by the RS.

Cooperative relaying is a promising extension to relay networks where several RSs transmit jointly to the same RN yielding diversity gain [3]. Due to the spatial separation of different RSs, cooperative relaying can be interpreted as distributed multiple antenna transmission. Orthogonal space-time block codes (OSTBC), which have been first proposed by Alamouti [4] for the case of two transmit antennas, exploit spatial diversity by using multiple transmit antennas [5]. Mietzner and Hoehrer [6] showed the applicability of the two antenna Alamouti code as a distributed OSTBC [7][8] in a cooperative relay network, i.e., the investigations in [6] are restricted to an OSTBC which is applied in a distributed fashion at two different RSs with one antenna each (single input single output (SISO) RSs).

In this paper, the performance and effort of two cooperating SISO RSs analysed in [6] is compared to the performance and effort of two cooperating RSs with two transmit antennas each (multiple input multiple output (MIMO) RSs) that apply a four antenna quasi-orthogonal space-time block code (Q-OSTBC) with constellation rotation [9] in a distributed fashion. Additionally, other arrangements of the overall four transmit antennas are considered by changing the number of cooperating RSs, e.g., four RSs with one transmit antenna each and the non-cooperative case of one RS with four transmit antennas, respectively.

For MIMO channels it is very likely that correlated channel fading coefficients appear if the transmit antennas of the same transmitter/receiver are within a range of a few wavelengths. This leads to a degradation of the bit error rate (BER) performance of non-distributed space-time block codes (STBCs) as diversity is lost. For spatially separated cooperating RSs it is less likely that the different channel coefficients are correlated. Nevertheless, for RSs with more than one antenna, correlation between adjacent antennas at the same RS still appears. Additionally, there appear different channel gains from the different RSs to the RN. Without transmit power control, a BER performance degradation due to the distributed fashion of the STBC is expected. In this paper, the BER performance degradation due to correlated channel coefficients as well as different channel gains is derived theoretically by extending the Chernoff bound, which is an upper bound of the BER in Rayleigh fading channels, by the correlation factor of adjacent antennas at the same RS and by channel gain factors modeling different receive powers from several cooperating RSs. The theoretical results are confirmed by means of simulations.

The paper is organized as follows: The basic principle of Q-OSTBC with constellation rotation is described in Section II. The system model of a cooperative relay network applying STBCs is derived in Section III. The different antenna arrangements in a relay network are introduced in Section IV. Section V gives a theoretical performance analysis of distributed STBCs which is confirmed by the simulation results in Section VI. Section VII finally concludes this work.

II. Quasi-Orthogonal Space-Time Block Codes

In this section, the principle of Q-OSTBCs with constellation rotation, which are later used as distributed codes in a cooperative relay network, is derived starting with OSTBCs [10]. Assuming T orthogonal time intervals and M transmit antennas, an orthogonal design for N complex
symbols $x(1), x(2), \ldots, x(N)$ is defined by a code matrix $C(x(1), x(2), \ldots, x(N))$ of dimension $T \times M$, with $TM \geq N$, such that

(i) the entries of $C$ are complex linear combinations of $x(1), x(2), \ldots, x(N)$ and their conjugate complexes $x(1)^*, x(2)^*, \ldots, x(N)^*$

(ii) $C^H C = \left( \sum_{n=1}^{N} |x(n)|^2 \right) I_M$

where $[.]^H$ designates the conjugate transpose and $I_M$ is a $M \times M$ identity matrix. The symbol transmission rate of these codes is defined as $I = \frac{\log_2 M}{TM}$. The loss of perfect orthogonality can be designed [9]. The loss of perfect orthogonality can be checked by

$$C^H C = \begin{bmatrix} a & 0 & b & 0 \\ 0 & a & 0 & -b \\ -b & 0 & a & 0 \\ 0 & b & 0 & a \end{bmatrix}$$

where

$$a = \sum_{n=1}^{4} |x(n)|^2$$

$$b = x(1)^* x(3) - x(1) x(3)^* - x(2)^* x(4) + x(2) x(4)^*.$$ (4)

Because of the inter-symbol-interference indicated by variable $b$ in (2), the BER performance of this Q-OSTBC is degraded. However, from equations (1) and (2) one notes that the code matrix can be decoupled into two sub-matrices with

$$C = C_1 (x(1), 0, x(3), 0) + C_2 (0, x(2), 0, x(4))$$

where $C_1^H C_2 + C_2^H C_1 = 0$ for all $x(n)$. For both symbol pairs $\{x(1), x(3)\}$ and $\{x(2), x(4)\}$, the ML decoding at the receiver can be processed independently. From literature, two different approaches for improving the BER performance of Q-OSTBCs are known, which are compared in the following. In [9], it is proposed to take $x(1)$ and $x(3)$ from different symbol constellations $A_1$ and $A_2 = e^{j\phi} A_1$, respectively, by rotating the constellation of $x(3)$ by an angle $\phi$. Similarly, $x(2)$ is taken from $A_1$ and $x(4)$ is taken from $A_2$ for the second symbol pair. By computer search, the optimum rotation angle $\phi_{opt}$ can be found under the constraint of maximizing the minimum Euclidean distance between all different representations of the symbol pairs $\{x(1), x(3)\}$ and $\{x(2), x(4)\}$, respectively. For symbols taken from a QPSK constellation, $\phi_{opt} \approx 0.525$ maximizes the minimum Euclidean distance.

Constellation rotation for Q-OSTBCs is also considered in [11]. However, another optimization approach is proposed which reestablishes orthogonality for Q-OSTBCs. There, it is shown for QPSK and all other QAM modulation schemes taken from a square lattice that $\phi_{opt,2} = \pi/4$ is the optimum rotation angle. To find out which optimization approach should be used in practice, the BER performance for both approaches is compared by means of simulations. Figure 1 depicts the BER depending on the rotation angle $\phi$ for different signal-to-noise ratios $E_s/N_0$ where $E_s$ denotes the average transmit energy per transmit symbol and $N_0$ the constant noise power density spectrum. The symmetry of the curves comes from the

![Figure 1: BER performance for constant $E_s/N_0$ with different rotation angles $\phi$ between two QPSK constellations.](image)

III. COOPERATIVE RELAYING SYSTEM MODEL

In this section, the system model for cooperative relaying applying distributed STBCs is introduced. For relaying, two orthogonal channel resources are required. By using the first channel resource, the SN transmits to $K$ RSs. Throughout
the whole paper, it is assumed that a direct communication between the SN and the RN is not possible, i.e., the RN receives no symbols from the SN. By using the second channel resource, the RSs retransmit a processed version of the previously received signals to the RN. It is assumed that there are $M_{RS}^{(k)}$ transmit antennas at $RS^{(k)}$, $k = 1, \ldots, K$, and $M_{RN}$ receive antennas at the RN.

Distributed STBCs are applied to groups of $N$ symbols in $T$ symbol intervals using the total number of transmit antennas $M_{RS} = \sum_{k=1}^{K} M_{RS}^{(k)}$ at $K$ RSs. At each $RS^{(k)}$, the received vector from the SN is processed according to the considered relaying concept (AF or DF) resulting in the symbol vector $r_{RS}^{(k)}$ of elements $r_{RS}^{(k)}(n)$, $n = 0, 1, \ldots, N$. The RSs store this symbol vector and retransmit a processed version with respect to the applied STBC. During $T$ time intervals a distributed STBC across all $K$ RSs, each with $M_{RS}^{(k)}$ transmit antennas, is applied, i.e., it is assumed that the RSs are synchronized and from each antenna a complex linear combination of the symbols $s_{RS}^{(k)}(n)$ and their conjugate complexes $s_{RS}^{(k)}(n)^*$ is transmitted according to the applied distributed STBC. The elements of the coded transmit matrix $R_{RS}^{(k)}$ of dimension $T \times M_{RS}^{(k)}$ at $RS^{(k)}$ are complex linear combinations of symbols $s_{RS}^{(k)}(n)$ and $s_{RS}^{(k)}(n)^*$. All coded transmit matrices $R_{RS}^{(k)}$ can be combined in the coded transmit matrix over all RSs

$$\mathbf{R}_{RS} = \left[ \mathbf{r}_{RS}^{(1)}, \ldots, \mathbf{r}_{RS}^{(K)} \right].$$

Note that for $K = 1$ with $M_{RS}$ transmit antennas at one RS, the combined code matrix is equal to the code matrix of the non-distributed STBC in (1), assuming $r_{RS}^{(1)}(n) = x(n)$. The $RS^{(k)}$-to-RN channel is described by matrix $\mathbf{H}^{(k)}$ of dimension $M_{RS}^{(k)} \times M_{RN}$. Let $E\{\cdot\}$, $tr\{\cdot\}$ and $[\cdot]^T$ denote the expectation, the sum of the main diagonal elements of a matrix and the transpose, respectively. Then, the average normalized channel gain of $\mathbf{H}^{(k)}$ is defined by $E\{tr\{\mathbf{H}^{(k)}\mathbf{H}^{(k)^T}\}\} = \alpha^{(k)} M_{RN} M_{RS}^{(k)}$, where $\alpha^{(k)}$ models different channel gains from each $RS^{(k)}$ to the RN under the constraint

$$\sum_{k=1}^{K} \alpha^{(k)} M_{RS}^{(k)} = M_{RS}.$$  

With Eq. (7), the overall channel matrix

$$\mathbf{H} = \left[ \mathbf{H}^{(1)^T}, \ldots, \mathbf{H}^{(K)^T} \right]^T$$

has a normalized average channel gain of $E\{tr\{\mathbf{HH}^T\}\} = M_{RN} M_{RS}$, i.e., the overall average channel gain stays constant while different RS-to-RN channels contribute different fractions of this channel gain which is modeled by $\alpha^{(k)}$. It is assumed that the overall transmit energy per transmit symbol $E_x$ at the RSs is shared equally among the $M_{RS}$ transmit antennas of all RSs. The overall receive matrix $\mathbf{R}_{RN}$ at the RN of dimension $T \times M_{RN}$ is a superposition of all single receive matrices from $K$ RSs after $T$ time intervals and results in

$$\mathbf{R}_{RN} = \sqrt{\frac{E_x}{M_{RS}}} \mathbf{R}_{RS} \mathbf{H} + \mathbf{N}_{RN}$$

where the elements of the noise matrix $\mathbf{N}_{RN}$ are zero mean complex Gaussian random variables with constant power density spectrum $N_0$.

IV. ANTENNA ARRANGEMENTS

In the following, all considerations are restricted to the case of $M_{RS} = 4$ transmit antennas distributed among different numbers of RSs. The Q-OSTBC with constellation rotation of (1) is applied in a distributed fashion at the cooperating RSs. Symbols $r_{RS}^{(k)}(3)$ and $r_{RS}^{(k)}(4)$ are taken from a QPSK constellation rotated by $\pi/4$ compared to the QPSK constellation of $r_{RS}^{(k)}(1)$ and $r_{RS}^{(k)}(2)$. Depending on $K$ and $M_{RS}^{(k)}$, there are five possible arrangements of four transmit antennas:

(i) all 4 antennas are at one RS, i.e., $K = 1$ and $M_{RS}^{(1)} = 4$

(ii) 4 RSs each with one antenna, i.e., $K = 4$ and $M_{RS}^{(k)} = 1$ for $k = 1, \ldots, 4$

(iii) 2 RSs each with 2 antennas, i.e., $K = 2$ and $M_{RS}^{(1)} = M_{RS}^{(2)} = 2$

(iv) 2 RSs, one RS with 3 antennas and one RS with 1 antenna, i.e., $K = 2$ and $M_{RS}^{(1)} = 3$, $M_{RS}^{(2)} = 1$

(v) 3 RSs, one RS with 2 antennas and two RSs with 1 antenna, i.e., $K = 3$ and $M_{RS}^{(1)} = 2$, $M_{RS}^{(2)} = M_{RS}^{(3)} = 1$

Although the overall average transmit energy $E_x$ per transmit symbol is equal in all five cases, a fair comparison between them can be difficult. In infrastructure relay networks, for example, the equipment costs are higher for establishing case (iii) than case (i). It is also less likely that one RN has good link conditions to four different RSs in case (ii) than to one RN in case (i).

V. PERFORMANCE ANALYSIS

Without transmit power control in case of antenna arrangements (ii) to (v) different symbols of the distributed STBC are received with different average powers for different average channel gains modeled by $\alpha^{(k)}$. This leads to a degradation of the BER performance, which is analysed in the following for case (iii). Additionally, the degradation of the BER performance due to correlated channel coefficients is considered. STBCs shall exploit transmit diversity. Hence, in order to investigate the diversity order of the coding scheme, it is sufficient to assume $M_{RN} = 1$ receive antenna. In this case, the channel matrix $\mathbf{H}^{(k)}$ reduces to a vector of channel coefficients $h^{(k)}(m)$, $m = 1, \ldots, M_{RS}$. The complex Gaussian channel coefficients from different RSs are assumed to be spatially uncorrelated. Channel coefficients $h^{(k)}(m)$ assigned to the same $RS^{(k)}$ are correlated and the channel matrix $\mathbf{H}^{(k)}$ is modeled by

$$\text{vec}\left\{\mathbf{H}^{(k)}\right\} = \mathbf{S}^{(k)} \text{vec}\left\{\mathbf{H}_c^{(k)}\right\}$$

where $\text{vec}\{\cdot\}$ stacks $\cdot$ into a column vector columnwise, $\mathbf{H}_c^{(k)}$ is spatially white and $\mathbf{S}^{(k)}$ is the $M_{RS}^{(k)} \times M_{RS}^{(k)}$ covariance matrix.
defined as
\[
S^{(k)} = \frac{1}{M^{(k)}_{\text{RS}}} \begin{bmatrix}
1 & \rho^1 & \cdots & \rho^{M^{(k)}_{\text{RS}}-1} \\
\rho^1 & 1 & \cdots & \rho^{M^{(k)}_{\text{RS}}-2} \\
\vdots & \vdots & \ddots & \vdots \\
\rho^{M^{(k)}_{\text{RS}}-1} & \rho^{M^{(k)}_{\text{RS}}-2} & \cdots & 1
\end{bmatrix}
\]
with correlation coefficient \(0 \leq \rho \leq 1\) and \(\rho = 0\) defining uncorrelated channel coefficients [12].

Assuming ML detection at the RN and applying the Chernoff bound for channel coefficients with Rayleigh distributed amplitudes, the average BER may be upper bounded by
\[
\text{BER} \leq \tilde{N}_e \left| \text{det} \left( I_{M_{\text{RS}}} + \frac{E_d d_{\text{min}}^2}{4M_{\text{RS}}N_0} \mathbf{W} \right) \right|^{-1}.
\]
[12], where \(\text{det}(...)\) denotes the determinant, \(\mathbf{W} = \text{E} \{ \text{vec} (\mathbf{H}) \text{vec} (\mathbf{H})^H \}\) is the covariance matrix of the overall channel and \(\tilde{N}_e\) and \(d_{\text{min}}\) are the number of nearest neighbours and minimum Euclidean distance in the constellation diagram, respectively.

Applying the correlated channel model in (10), the upper BER bound of (12) in the high SNR regime for case (iii) may be described by
\[
\text{BER} \leq \tilde{N}_e \left( \frac{E_d d_{\text{min}}^2}{16N_0} \right)^{-1} \left( \alpha^{(1)} \alpha^{(2)} (1 - \rho) \right)^{-2}.
\]

In case (iii), applying Eq. (7) leads to \(\alpha^{(1)} + \alpha^{(2)} = 2\), i.e., for \(\alpha^{(1)} = 1\), \(\text{RS}^{(1)}\) and \(\text{RS}^{(2)}\) are received with the same average power and for \(\alpha^{(1)} = 0\) the whole channel gain comes from \(\text{RS}^{(2)}\) as \(\text{RS}^{(1)}\) fails completely. Eq. (13) shows that for totally uncorrelated channel coefficients (\(\rho = 0\)) and equal channel gains from both RSs (\(\alpha^{(1)} = 1\)), the BER performance of the non-distributed STBC with a diversity order of 4 is achieved [12]. With increasing correlation coefficient \(\rho\) between adjacent antennas and different channel gains \(\alpha^{(k)}\), the BER performance is degraded which is indicated by the BER degradation factor
\[
\beta_{\text{deg}} = \left( \alpha^{(1)} \alpha^{(2)} (1 - \rho) \right)^{-2} \geq 1.
\]

Figure 2 shows the increase of BER degradation factor \(\beta_{\text{deg}}\) for decreasing \(\alpha^{(1)}\) with \(\rho = 0\) and for increasing \(\rho\) with \(\alpha^{(1)} = 1\), respectively. It can be seen that the slope of \(\beta_{\text{deg}}\) for decreasing \(\alpha^{(1)}\) is lower than for increasing \(\rho\). Hence, the BER performance is less sensitive to different channel gains than to correlated channel coefficients. This observation is also confirmed by the following simulation results.

VI. SIMULATION RESULTS

In this section, the characteristics of cooperative MIMO relay networks are presented by means of simulations for the extreme antenna arrangement cases (i), (ii), and (iii). Cases (iv) and (v) are omitted since they provide no essentially new characteristics. For simplicity, perfect SN-to-RS links are assumed as the paper focuses on the cooperation between the RSs on the RS-to-RN links. At the RN, ML decoding is applied assuming perfect channel knowledge. The receive signals from different cooperating RSs are perfectly synchronized in time.

Figure 3 shows the BER performance for two cooperating RSs with different channel gains when successively decreasing \(\alpha^{(1)}\). For this figure, it is assumed that the channel coefficients as-

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cessing effort as the ML decoding has to be processed jointly for two transmit symbols in case of Q-OSTBC.

In Fig. 4, equal channel gain is assumed for the different antenna arrangement cases (i) to (iii) with different correlation coefficients $\rho$ for the MIMO channels $H^{(k)}$. In case (ii), the best BER performance is achieved since all channel coefficients from the 4 RSs to the RN are uncorrelated due to the distributed arrangement of the single transmit antennas. For increasing correlation coefficient $\rho$, the BER performance of cases (i) and (iii) shows an obvious degradation. However, in the cooperative relaying case (iii) there are only two pairs of correlated transmit antennas while these two pairs are mutually uncorrelated. Hence, the performance of case (iii) is still better than for non-distributed STBCs at one RS with four spatially correlated transmit antennas, e.g., for $\rho = 0.6$ and high $E_s/N_0$ the performance of case (iii) is about 3dB better than the performance of case (i).

In Fig. 5, the correlation coefficient is set to $\rho = 0.6$ and the single RS case (i) and the cooperative relaying case (iii) for different channel gains $\alpha^{(1)}$ on the RS$^{(1)}$-to-RN link are compared to each other. On the one hand, the performance degrades with decreasing receive power from RS$^{(1)}$ in case (iii). But on the other hand, it is worth noting that even in the case of 10dB receive power loss ($\alpha^{(1)} = 0.1$) from RS$^{(1)}$ the performance of cooperative relaying is still about 1dB better than for the single RS with 4 transmit antennas in the high $E_s/N_0$ regime. In case of different channel gains, distributed STBCs show less BER performance degradation than non-distributed STBCs in case of correlated channel coefficients.

VII. Conclusion

In this paper, the application of a distributed four antenna Q-OSTBC with constellation rotation for cooperative relay networks is considered. It is shown that two cooperating MIMO RSs achieve a better BER performance than two cooperating SISO RSs at the cost of additional transmit antennas and higher decoding effort at the receiver. Applying distributed STBCs in cooperative relay networks reduces the probability of correlated channel coefficients as the RSs are spatially separated. It is shown that even in case of different receive powers from several cooperating RSs at the receiver, the performance of cooperative relaying is better than the performance of non-distributed STBCs in spatially correlated MIMO channels.

REFERENCES


