Downlink beamforming and SINR balancing for the simultaneous provision of unicast/multicast services

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***Abstract*—The provision of multicast services is a relevant feature within the context of the further evolution of cellular communication systems. The scope of this paper lies on the application of adaptive antenna arrays in order to allow for spatial division multiple access (SDMA) in a scenario with both unicast and multicast users. A sub-optimum algorithmic solution is proposed for the problem of maximizing the minimum SINR perceived by the unicast/multicast users (SINR balancing), while satisfying the transmit power constraint. It is based on the unicast-only solution, which has been extended to the unicast/multicast case. It is shown that the proposed low-complexity algorithm provides a reasonable approximation to the optimal solution, outperforming diagonalization-based procedures such as block diagonalization.**

I. Introduction

The provision of multicast/broadcast services is an important feature within the context of the further evolution of cellular communication systems [1]. Services whose content is targeted at multiple users within the system, such as audio/video streaming, mobile TV, localized services, among others, may be implemented over point-to-multipoint connections. The use of such connections spares resources and is spectrally efficient. Nevertheless, they make it more difficult to adapt to the channel conditions of each specific user.

Considering the downlink of a wireless cellular system, adaptive antenna arrays may be employed at the base station in order to improve the quality perceived by the users within multicast groups, which has been investigated by previous work [2-5]. The implementation of spatial division multiple access (SDMA), so that multiple unicast and multicast users may share the same radio channel, is a further measure for increasing spectral efficiency. In [6], a solution based on semidefinite relaxation has been proposed for the problem of minimizing the transmit power, subject to providing a certain target signal-to-interference-plus-noise ratio (SINR) to the users. In [7], a different approach based on block-diagonalization has been proposed for eliminating the interference among the groups and maximizing the worst-user SINR.

In this paper the focus lies on a new sub-optimum solution for the problem of maximizing the minimum SINR perceived by the unicast/multicast users (SINR balancing), while satisfying the transmit power constraint. The semidefinite optimization methodology of [6, 8] cannot be directly applied to this problem, since the maximum achievable worst-user SINR would have to be known a priori. Also, differently from [7], no diagonalization is performed, and therefore the number of users is no longer limited by the number of antenna elements.

The proposed algorithm is based on the solution for the unicast-only case [9], which divides the problem into power allocation and unit-norm beamforming, and employs alternating optimization. It is shown that some approximations can be done for the unicast/multicast case, which still allow the power allocation and unit-norm beamforming to be efficiently solved as eigenvalue problems. In order to further improve the SINR balancing, a final power redistribution step is also proposed.

Some of the terms employed throughout the paper may have multiple interpretations. Therefore, in order to avoid misunderstanding, they are defined as follows. The complete beamforming problem corresponds to the determination of the weight vectors of all users according to a certain criterion. This problem can also be divided in two parts: unit-norm beamforming, where the unit-norm weight vector of each user is determined, and power allocation, which determines the amount of power allocated to each of these vectors. The term SINR balancing refers to the maximization of the minimum SINR perceived by the users, and it corresponds to the effect that the proposed beamforming algorithm aims to achieve.

The paper is organized as follows. In section II the system model is presented. Section III briefly describes the block diagonalization algorithm for comparison purposes. Section IV presents the proposed new beamforming algorithm for achieving the SINR balancing. Section V analyzes the performance results, and, finally, conclusions are drawn in section VI.

II. System model

A multi-user multi-carrier system is considered which assumes flat-fading per sub-carrier and negligible inter-symbol interference, so that the data symbols can be treated individually. The system model corresponds to the downlink of a single cell in a cellular system containing both unicast and multicast users. The base station is equipped with an M-element antenna array, while the N mobile stations are single-antenna devices. Considering a vector $\mathbf{d} \in \mathbb{C}^N$ with N data symbols, which are modulated by a matrix $\mathbf{M} \in \mathbb{C}^{M\times N}$, transmitted over the radio channel $\mathbf{H} \in \mathbb{C}^{N\times M}$, subject to additive white Gaussian noise $\mathbf{n} \in \mathbb{C}^N$, and demodulated by a matrix $\mathbf{D} \in \mathbb{C}^{N\times N}$, the N downlink estimates $\hat{\mathbf{d}} \in \mathbb{C}^N$ of the N transmitted symbols $\mathbf{d}$ may be written as

$$\hat{\mathbf{d}} = \mathbf{DHMd} + \mathbf{Dn}. \tag{1}$$

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The N users are assumed to be divided into K multicast groups. The number of users within each group is represented by vector $\mathbf{g}_{K\times 1}$, whose k^{th} element $g_k \in \{1,\ldots,N\}$ indicates the number of users within group k. Note that the unicast users can be interpreted as multicast groups of unit size and that $\sum_{k=1}^{K} g_k = N$. In order to associate which users belong to which group, an index vector $\mathbf{b}_{N\times 1}$ is also introduced, whose n^{th} element $b_n \in \{1,\ldots,K\}$ indicates the group to which user n belongs. For example, in a system with two unicast users and one multicast group composed of two users, we would have: $N=4$, $K=3$, $\mathbf{g}=[1,1,2]^T$, and $\mathbf{b}=[1,2,3,3]^T$, with $(\cdot)^T$ denoting the transpose operator.

Since the users of a multicast group expect the same symbol, the number K of multicast groups is also equivalent to the number of data streams. For this reason there are $N-K$ repeated entries within vector $\mathbf{d}\in\mathbb{C}^N$. The removal of such entries results in vector $\mathbf{d}'\in\mathbb{C}^K$. This operation can be mathematically expressed as $\mathbf{d}'=\mathbf{T}\mathbf{d}$, where $\mathbf{T}\in\mathbb{R}_+^{K\times N}$ is a transformation matrix with the n^{th} column given by $\mathbf{t}_n = g_{b_n}^{-1}\mathbf{e}_{b_n}$, for which $\mathbf{e}_i$ corresponds to the i^{th} column of the identity matrix of dimension K. The reduced dimension of the data vector also leads to a reduced modulation matrix $\mathbf{M}'\in\mathbb{C}^{M\times K}$, i.e., instead of one beamforming vector per user there is now one beamforming vector per multicast group. Let $\mathbf{m}_i$ and $\mathbf{m}'_i$ represent the i^{th} column of matrices $\mathbf{M}$ and $\mathbf{M}'$, respectively, they are related by

$$\mathbf{m}'_k = \sum_{n\in\mathcal{N}_k} \mathbf{m}_n\,, \quad \text{for} \quad k=1,\ldots,K\,, \tag{2}$$

where $\mathcal{N}_k$ corresponds to the set of users which belong to group k, i.e., for which $b_n = k$. After substituting $\mathbf{M}'$ and $\mathbf{d}'$, equation (1) can be rewritten in reduced form as

$$\hat{\mathbf{d}} = \mathbf{D}\mathbf{H}\mathbf{M}'\mathbf{d}' + \mathbf{D}\mathbf{n}. \tag{3}$$

Since independent single antenna users are considered, the demodulation matrix $\mathbf{D}$ is diagonal, and it is assumed that a matched filter is implemented at each receiver. The diagonal elements $D_{n,n}$ of matrix $\mathbf{D}$, considering that $\mathbf{h}_n$ corresponds to the n^{th} row of $\mathbf{H}$, can be expressed as

$$D_{n,n} = \frac{(\mathbf{h}_n\mathbf{m}'_{b_n})^*}{|\mathbf{h}_n\mathbf{m}'_{b_n}|^2}\,, \quad \text{for} \quad n=1,\ldots,N\,, \tag{4}$$

where $(\cdot)^*$ denotes the complex conjugate, and $|\cdot|$ is the absolute value operator.

The channel spatial covariance matrix $\mathbf{R}_n\in\mathbb{C}^{M\times M}$ of each user n is given by $\mathbf{R}_n = \mathrm{E}\{\mathbf{h}_n^H\mathbf{h}_n\}$, where $\mathrm{E}\{\cdot\}$ denotes the expectation operator and $(\cdot)^H$ is the conjugate transpose. The expression for the downlink SINR γ_n, assuming an average noise power of σ_n^2, is given by

$$\gamma_n = \frac{\mathbf{m}'^H_{b_n}\mathbf{R}_n\mathbf{m}'_{b_n}}{\sum\limits_{k=1,\,k\neq b_n}^{K} \mathbf{m}'^H_k\mathbf{R}_n\mathbf{m}'_k + \sigma_n^2}, \quad \text{for} \quad n=1,\ldots,N\,. \tag{5}$$

It is assumed that the maximum power available for transmission is denoted by P. As a consequence, the design of matrix $\mathbf{M}$ must satisfy the following energy constraint: $\mathrm{trace}(\mathbf{M}^H\mathbf{M}\mathbf{R}_d) \leq P$, with the signal correlation matrix defined as $\mathbf{R}_d = \mathrm{E}\{\mathbf{d}\mathbf{d}^H\}\in\mathbb{C}^{N\times N}$. Note that $\mathbf{R}_d$ converges to a block diagonal matrix, with each block k equal to $\mathbf{1}_{g_k\times g_k}$, which corresponds to a square matrix composed of ones. Equivalently, the constraint may be expressed as: $\mathrm{trace}(\mathbf{M}'^H\mathbf{M}'\mathbf{R}'_d)\leq P$, for which $\mathbf{R}'_d = \mathrm{E}\{\mathbf{d}'\mathbf{d}'^H\}\in\mathbb{C}^{K\times K}$ converges to $\mathbf{I}_K$, which is an identity matrix of dimension K.

III. Block diagonalization

The block diagonalization algorithm applied to this unicast/multicast scenario aims at maximizing the minimum SINR, subject to the condition that no interference should be perceived among the different groups. The optimization problem can be written as

$$\begin{aligned} &\mathbf{M}_{\text{BD}} = \underset{\mathbf{M}}{\operatorname{argmax}}\, \min_n \gamma_n\,, \quad \text{for} \quad n=1,\ldots,N\,,\\ &\text{subject to:}\\ &\begin{cases} \mathrm{trace}(\mathbf{M}^H\mathbf{M}\mathbf{R}_d)\leq P,\\ \mathbf{h}_i\mathbf{m}_j = 0, \quad \forall\, i,j\in\{1,\ldots,N\}\,|\,b_i\neq b_j. \end{cases} \end{aligned} \tag{6}$$

A solution to this problem within the unicast/multicast context has been proposed in [7], and it results in the following modulation matrix:

$$\mathbf{M}_{\text{BD}} = \beta\mathbf{N}\mathbf{B}\mathbf{\Gamma}, \tag{7}$$

where $\beta\in\mathbb{R}_+$ is an energy normalization factor and $\mathbf{N}\in\mathbb{C}^{N\times N}$ is responsible for performing the block diagonalization itself [10], such that $\mathbf{H}\mathbf{N}$ is a block-diagonal matrix. The matrices $\mathbf{B}\in\mathbb{C}^{N\times N}$ and $\mathbf{\Gamma}\in\mathbb{C}^{N\times N}$ are block-diagonal, so that they can separately process the multicast groups. The former performs multicast beamforming within each group and the latter distributes the power among the different groups.

IV. SINR balancing

The SINR balancing algorithm has the purpose of maximizing the minimum SINR perceived by the users. The difference with regard to block diagonalization is that interference is now tolerated and the number of users is no longer upper-limited by the number of antennas. The optimization problem is similar to (6), but without the zero-interference constraint:

$$\begin{aligned} &\mathbf{M}_{\text{SB}} = \underset{\mathbf{M}}{\operatorname{argmax}}\, \min_n \gamma_n\,, \quad \text{for} \quad n=1,\ldots,N\,,\\ &\text{subject to: } \mathrm{trace}(\mathbf{M}^H\mathbf{M}\mathbf{R}_d)\leq P\,. \end{aligned} \tag{8}$$

The semidefinite optimization methodology of [6, 8] cannot be directly applied to this problem, since the maximum achievable worst-user SINR would have to be known a priori. In the case of unicast-only users, an algorithmic solution for this problem has been presented in [9]. It takes advantage of the uplink/downlink duality and consists of an alternating optimization procedure, which adjusts both the unit-norm beamformers and the power allocation among the streams, converging to the optimal solution after only a few iterations.

In this paper we propose a modified version of the algorithm in [9], such that the solution to the unicast/multicast case can be approximated. It requires that the problem of finding $\mathbf{M}$ within (8) be separated into the power allocation and unit-norm beamforming procedures and that some assumptions be made. Let $\tilde{\mathbf{R}}_n = \mathbf{R}_n/\sigma_n^2 \in \mathbb{C}^{M\times M}$ denote the normalized channel covariance matrix of user n. Let $\mathbf{p} \in \mathbb{R}_+^N$ represent the power allocation vector, with each element p_n denoting the power allocated to user n, and $\mathbf{u}_n \in \mathbb{C}^M$ the unit-norm beamforming vector associated to user n, such that $p_n = ||\mathbf{m}_n||^2$, $\mathbf{u}_n = \mathbf{m}_n/||\mathbf{m}_n||$, and $\mathbf{U} = [\mathbf{u}_1 \ldots, \mathbf{u}_N] \in \mathbb{C}^{M\times N}$.

The algorithm is described in the following subsections. First the power allocation procedure for a fixed matrix $\mathbf{U}$ is presented, followed by the unit-norm beamforming given a fixed power allocation, and finally a power redistribution step is introduced in order to balance the SINRs among the unicast users and multicast groups.

A. Power allocation

In order to express the set of equations that determines the downlink power assignment given a fixed matrix $\mathbf{U}$, it is initially assumed that all users are unicast and that they achieve the same maximum SINR γ_{max}. Let $\mathbf{S} \in \mathbb{C}^{N\times N}$ denote a diagonal matrix corresponding to the signal part of the transmission, and $\mathbf{\Psi} \in \mathbb{C}^{N\times N}$ the interference part, such that for the unicast-only case the elements of $\mathbf{S}$ and $\mathbf{\Psi}$ are

$$S_{i,i} = \mathbf{u}_i^H \tilde{\mathbf{R}}_i \mathbf{u}_i\,, \qquad \Psi_{i,j} = \begin{cases} 0, & i = j \\ \mathbf{u}_j^H \tilde{\mathbf{R}}_i \mathbf{u}_j, & i \neq j \end{cases}. \tag{9}$$

According to [9] the unicast power allocation vector may be found by solving the following system of equations

$$\begin{cases} \gamma_{max}^{-1}\,\mathbf{p} = \mathbf{S}\mathbf{\Psi}\mathbf{p} + \mathbf{S}\mathbf{1} \\ \mathbf{1}^T\mathbf{p} = P \end{cases}, \tag{10}$$

where $\mathbf{1}$ is a vector of ones with appropriate dimension. The solution to the power allocation problem is obtained after expressing (10) as an eigensystem, and the power allocation vector is set to be the eigenvector associated to the largest eigenvalue λ_{max} of the corresponding coupling matrix [9].

For the combined unicast/multicast case this procedure cannot be directly applied, since the power allocation would have to be done for each group, and not for each user. This results in a number of equations larger than the number of variables, i.e., there are still N SINR values to balance but only K power elements to adjust. In this case it is not always possible to guarantee that all users achieve the same SINR and the system cannot be solved as an eigenvalue problem.

In order to simplify the problem and allow the unicast/multicast case to be also expressed by equation (10), it is here assumed that the power allocation can be done user-wise, i.e., vector $\mathbf{p}$ contains N elements, and the elements of matrices $\mathbf{S}$ and $\mathbf{\Psi}$ are defined as:

$$\begin{aligned} S_{i,i} &= \Big(\sum_{l\in\mathcal{N}_{b_i}} \mathbf{u}_l^H\Big)\tilde{\mathbf{R}}_i\Big(\sum_{l\in\mathcal{N}_{b_i}} \mathbf{u}_l\Big)\,, \\ \Psi_{i,j} &= \begin{cases} 0, & b_i = b_j \\ \mathbf{u}_j^H \tilde{\mathbf{R}}_i \mathbf{u}_j, & b_i \neq b_j \end{cases}. \end{aligned} \tag{11}$$

Matrices $\mathbf{S}$ and $\mathbf{\Psi}$ are chosen so that they approximate the actual SINR perceived by the users, while still allowing the system to be solved as an eigenvalue problem. The actual and approximate SINR expressions, respectively, are given by:

$$\begin{aligned} \gamma_n &= \frac{\Big(\sum_{l\in\mathcal{N}_{b_n}} \sqrt{p_l}\mathbf{u}_l^H\Big)\tilde{\mathbf{R}}_n\Big(\sum_{l\in\mathcal{N}_{b_n}} \sqrt{p_l}\mathbf{u}_l\Big)}{\sum_{k=1,\,k\neq b_n}^{K}\Big(\sum_{l\in\mathcal{N}_k} \sqrt{p_l}\mathbf{u}_l^H\Big)\tilde{\mathbf{R}}_n\Big(\sum_{l\in\mathcal{N}_k} \sqrt{p_l}\mathbf{u}_l\Big) + 1}\,, \\ \gamma_n &\simeq \frac{p_n\Big(\sum_{l\in\mathcal{N}_{b_n}} \mathbf{u}_l^H\Big)\tilde{\mathbf{R}}_n\Big(\sum_{l\in\mathcal{N}_{b_n}} \mathbf{u}_l\Big)}{\sum_{k=1,\,k\neq b_n}^{K}\Big(\sum_{l\in\mathcal{N}_k} p_l\mathbf{u}_l^H\tilde{\mathbf{R}}_n\mathbf{u}_l\Big) + 1}. \end{aligned} \tag{12}$$

The approximation of the signal part in (12) corresponds to considering the power of only the n^{th} user and disregarding the power of the other users belonging to the same group. With regard to the interference part, it is a worst-case approximation which considers all interferers as unicast users, instead of taking into account the equivalent group beamforming vectors.

B. Beamforming

Given a fixed power allocation, it has been shown in [9] for the unicast case that, due to the uplink/downlink duality, the optimal unit-norm beamformers can be obtained by performing maximization of the uplink SINR of each user independently. A similar approach is here considered for approximating the unicast/multicast case, and the optimization problem for the unit-norm beamformer of user n is written as

$$\begin{aligned} &\mathbf{u}_{n,\text{opt}} = \underset{\mathbf{u}_n}{\operatorname{argmax}} \frac{\mathbf{u}_n^H\tilde{\mathbf{R}}_n\mathbf{u}_n}{\mathbf{u}_n^H\mathbf{Q}_n\mathbf{u}_n}\,, \;\; \text{subject to: } ||\mathbf{u}_n||^2 = 1\,, \\ &\text{with } \mathbf{Q}_n = \sum_{k=1,\,k\neq b_n}^{K}\Big(\sum_{l\in\mathcal{N}_k} q_l\tilde{\mathbf{R}}_l\Big) + \mathbf{I}\,, \end{aligned} \tag{13}$$

where $\mathbf{q} \in \mathbb{R}_+^N$ represents the uplink power allocation vector, which may be obtained by the procedure described in subsection IV-A with the interference matrix transposed [9], i.e., $\mathbf{\Psi}^T$ instead of $\mathbf{\Psi}$ within (10). The solution of (13) corresponds to the dominant generalized eigenvector of the pair $(\tilde{\mathbf{R}}_n, \mathbf{Q}_n)$.

The difference with regard to the unicast-only case lies in the definition of matrix $\mathbf{Q}_n$, which has been modified to avoid interference within a same multicast group.

C. Iterative algorithm

The algorithm consists of the alternating optimization of the power allocation and unit-norm beamforming procedures, such as described in [9]. The dominant eigenvalue λ_{max} of the power allocation problem monotonically decreases after each iteration, so that the stop criterion is defined based on λ_{max} reaching a certain precision ϵ, i.e., $\lambda_{\text{max}}^{(i-1)} - \lambda_{\text{max}}^{(i)} < \epsilon$, where $(\cdot)^{(i)}$ indicates the i^{th} iteration. Given an arbitrary initial uplink power vector $\mathbf{q}^{(0)}$, the following steps are repeated until the desired precision is reached:

- Calculate $\mathbf{U}^{(i)}$ given the previous vector $\mathbf{q}^{(i-1)}$,
- Calculate $\mathbf{q}^{(i)}$ given matrix $\mathbf{U}^{(i)}$.

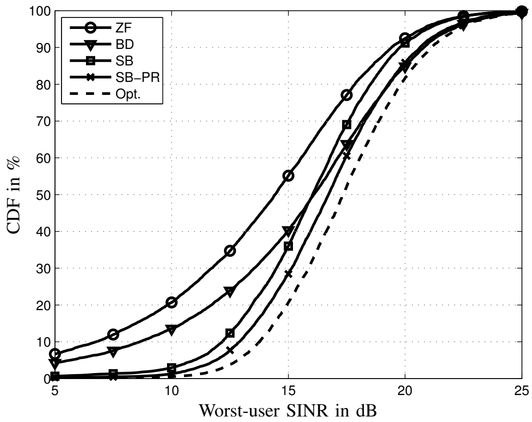

Fig. 1. CDF of the worst-user SINR for $M = 8$, $N_{\text{uc}} = 5$, and $N_{\text{mc}} = 3$.

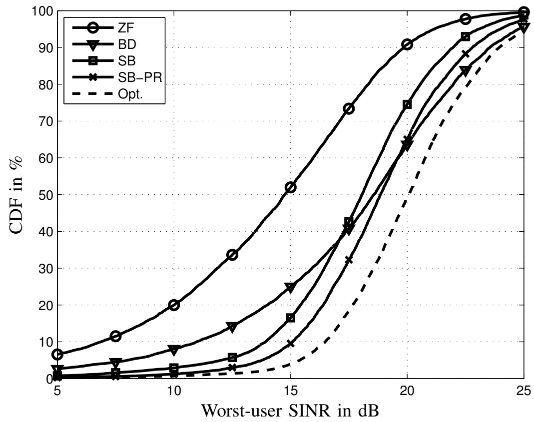

Fig. 2. CDF of the worst-user SINR for $M = 8$, $N_{\text{uc}} = 3$, and $N_{\text{mc}} = 5$.

At the end, the downlink power allocation $\mathbf{p}$ is calculated for the final matrix $\mathbf{U}$. The resulting modulation matrix is given by

$$\mathbf{M}_{\text{SB}} = \beta\,\mathbf{U}\,\text{diag}(\sqrt{\mathbf{p}})\,, \tag{14}$$

where $\beta \in \mathbb{R}_+$ is a normalization factor related to the total transmit power constraint, diag$(\cdot)$ returns a diagonal matrix containing the elements of the argument, and the $\sqrt{(\cdot)}$ operator is assumed to be applied to each element of a vector.

D. Power redistribution

Due to the SINR approximation considered for the power allocation procedure in section IV-A, the SINR balancing is not achieved for all users. In fact, it is perceived that the SINR of the unicast users reaches a certain balanced level, and that the average SINR of the users of the multicast group also approaches this level, but not each individual multicast user. In order to improve the worst-user performance, a power redistribution among the multicast and unicast users is proposed here. This procedure is an optional refinement of the algorithm presented in section IV-C, and is performed only a single time after the iterative algorithm has stopped.

The modulation matrix obtained by (14) can be expressed in its reduced form $\mathbf{M}'$ by employing (2). Let $\mathbf{p}' \in \mathbb{R}_+^K$ represent the group power allocation vector and $\mathbf{u}'_k \in \mathbb{C}^M$ the unit-norm beamforming vector of group k, such that $p'_k = ||\mathbf{m}'_k||^2$, $\mathbf{u}'_k = \mathbf{m}'_k/||\mathbf{m}'_k||$, and $\mathbf{U}' = [\mathbf{u}'_1, \ldots, \mathbf{u}'_K] \in \mathbb{C}^{M\times K}$. The users with lowest SINR are selected to represent each group, such that $\tilde{\mathbf{R}}'_k = \tilde{\mathbf{R}}_{n\,|\,\gamma_n=\min\gamma_{\mathcal{N}_k}}$, where $\gamma \in \mathbb{R}_+^N$ corresponds to the SINR vector that results from the application of $\mathbf{M}_{\text{SB}}$.

The unit-norm beamforming vectors $\mathbf{u}'_k$ are maintained and the power vector $\mathbf{p}'$ is recalculated by solving the system:

$$\begin{cases} \gamma_{max}^{-1}\,\mathbf{p}' = \mathbf{S}'\mathbf{\Psi}'\mathbf{p}' + \mathbf{S}'1 \\ \mathbf{1}^T\mathbf{p}' = P \end{cases}, \tag{15}$$

for which the elements of $\mathbf{S}' \in \mathbb{C}^{K\times K}$ and $\mathbf{\Psi}' \in \mathbb{C}^{K\times K}$ are given by

$$S'_{i,i} = \mathbf{u}_i'^H\tilde{\mathbf{R}}'_i\mathbf{u}'_i\,, \qquad \Psi'_{i,j} = \begin{cases} 0, & i = j \\ \mathbf{u}_j'^H\tilde{\mathbf{R}}'_i\mathbf{u}'_j, & i \neq j \end{cases}. \tag{16}$$

The solution of the system results in the power re-allocation vector, which is denoted $\mathbf{p}'_{\text{PR}}$. It is applied to the unit-norm beamforming, without the need of further power normalization, such that the new modulation matrix is given by

$$\mathbf{M}'_{\text{SB-PR}} = \mathbf{U}'\,\text{diag}\left(\sqrt{\mathbf{p}'_{\text{PR}}}\right). \tag{17}$$

V. Performance analysis

In this section, the performance of the different algorithms is evaluated. The system consists of a single cell serving a certain number N of users. It is assumed that there are N_{uc} unicast users and one multicast group containing N_{mc} users.

The users are uniformly distributed over one hexagonal sector of a tri-sectorized cell and a base station with an M-element antenna array is located at the sector corner. The considered propagation effects include the distance-based path-loss attenuation (with exponent $\alpha = 3.5$), as well as uncorrelated Rayleigh fading, which is modelled as circularly symmetric complex Gaussian random variables with variance σ^2. The path-loss is modelled by assuming that the cell border is at a distance $d_b = 1$ from the base station and that the fading variance of a user with distance $d \leq d_b$ is given by $\sigma^2 = 1/d^\alpha$ [2]. Additive white Gaussian noise is also assumed and the transmit power is adjusted to provide an average SNR of 10dB at the cell border.

In order to identify the algorithms throughout the performance analysis, the block diagonalization algorithm of section III is termed BD, the SINR balancing algorithm of section IV-C is termed SB, and the SINR balancing with the additional power redistribution of section IV-D is termed SB-PR. Additionally, the following two strategies are also considered by the evaluation: zero-forcing (ZF) [5], and the optimal solution (Opt.) of the SINR balancing problem obtained through a numerical optimization method (sequential quadratic programming).

Figs. 1 and 2 show the cumulative distribution function (CDF) of the worst-user SINR among all groups for a scenario with an 8-element antenna array. Fig. 1 depicts a situation in which there is a predominance of unicast users, while Fig. 2 shows the opposite case.

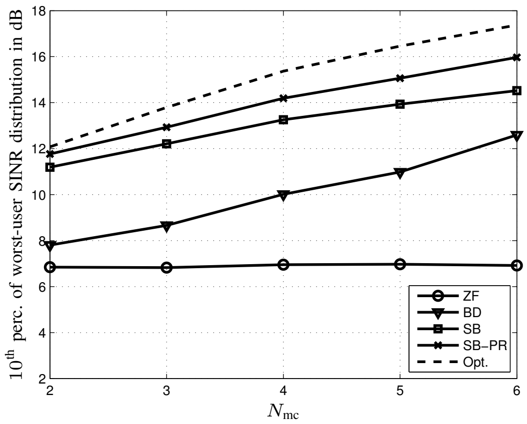

Fig. 3. Impact of the group sizes on the SINR for $M = 8$, $N_{uc} = M - N_{mc}$.

From both figures it can be seen that ZF presents the worst performance, as expected, since it unnecessarily performs interference cancellation within the multicast group. The BD algorithm presents some gains with regard to ZF, with its average performance approaching that of the other algorithms, but in terms of the 10^{th} percentile it is still significantly behind. Next, it can be seen that the proposed SB and SB-PR algorithms come closer to the optimal performance.

By comparing both figures it can be seen that Fig. 2 presents in general higher SINR values, even though the total number of users and antenna elements is the same. The reason for this behavior is that Fig. 2 has more multicast users, and therefore the intra-cell interference levels are lower. As a consequence of this interference reduction, the BD performance is significantly improved from Fig. 1 to Fig. 2. It can also be seen that the power redistribution procedure has a significant impact on the performance of the proposed SINR balancing algorithm, especially for the case in which there are more multicast users.

The impact of the different user configurations on the performance of the algorithms can be seen in Fig. 3. The performance is measured in terms of the 10^{th} percentile of the worst-user SINR distribution. The total number of users is kept equal to the number of antenna elements, but the proportion between unicast and multicast users is varied.

The relative behavior among the algorithms is the same as that observed for Figs. 1 and 2. It can be seen that as the proportion of multicast users increases, the SINR increases as well, for the same reasons already previously mentioned. The ZF algorithm is an exception, presenting practically no improvements, which is due to the fact that it treats unicast and multicast users without distinction.

It is also seen from Fig. 3 that SB-PR is the algorithm which best approximates the optimal solution for all evaluated user configurations. The difference between SB-PR and the optimal solution slightly increases for larger multicast groups. This is mainly due to the SINR approximation that was taken into account within section IV-A, which was implemented in order to allow for a low-complexity solution of the power allocation and unit-norm beamforming problems.

VI. Conclusions

This paper investigates the application of adaptive antenna arrays and SDMA techniques to the downlink of wireless systems containing both unicast and multicast users.

A sub-optimal algorithmic solution has been proposed for the problem of maximizing the minimum SINR perceived by unicast and multicast users (SINR balancing), while satisfying the power constraint. The proposed algorithm is based on the unicast-only solution, which has been extended for the unicast/multicast case, and consists of the alternate optimization of the power allocation and unit-norm beamforming problems. Additionally, a power redistribution step has been suggested in order to improve the performance of the algorithm.

The results have been presented in terms of the worst-user SINR, and different algorithms have been compared. The diagonalization-based algorithms (ZF and BD) present the worst performance, while the proposed algorithm (SB) approaches the solution of the SINR balancing problem obtained through numerical optimization. It could also be seen that the power redistribution (SB-PR) has a significant impact on the SINR balancing, resulting in higher worst-user SINR values.

The analysis of the results indicates that SB-PR reasonably approximates the optimal solution for all user configurations. In comparison to ZF and BD, the SB-PR algorithm also has the advantage of not requiring that the number of users be limited by the number of antennas. The difference between SB-PR and the optimal solution slightly increases for larger multicast groups, but note that SB-PR is a low-complexity algorithm, while a numerical solution obtained through quadratic optimization methods can be quite time-consuming.

An interesting topic for further studies consists of the investigation of efficient SDMA grouping techniques adequate for this unicast/multicast scenario, which might avoid situations in which there is a high correlation among the users' channels and therefore improve system performance.

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