APPLYING RELAY STATIONS WITH MULTIPLE ANTENNAS IN THE ONE- AND TWO-WAY RELAY CHANNEL

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ABSTRACT
This paper considers the two-hop relaying case with bi-directional communication of two nodes $S_1$ and $S_2$ via an intermediate relay station (RS). The RS is equipped with multiple antennas and channel state information is available at the RS. Either the multiple antennas can be used to achieve spatial diversity by applying receive and transmit maximum ratio combining (MRC) at the RS in a one-way relaying approach where up- and downlink are transmitted on orthogonal channel resources, or they can be used to apply the recently introduced multiple input multiple output (MIMO) two-way relaying. In MIMO two-way relaying, the number of required channel resources is reduced since up- and downlink are transmitted on the same channel resources. In this paper, it is investigated which approach provides a better average and outage performance, respectively. Concerning the average performance, MIMO two-way relaying always outperforms MRC one-way relaying. However, the outage performance in MIMO two-way relaying significantly depends on the choice of the linear filter at the RS. MIMO two-way relaying with a linear zero forcing filter provides a worse outage performance than MRC one-way relaying while MIMO two-way relaying with a linear minimum mean square error filter outperforms MRC one-way relaying.

I. INTRODUCTION
There exists much ongoing work in the field of relay networks [1] [2] [3]. This paper considers the two-hop relaying case where two nodes $S_1$ and $S_2$ exchange data via an intermediate relay station (RS) assuming that a direct communication between the two nodes is not possible, e.g., due to shadowing or limited transmit power. In two-hop relaying, the RS requires two orthogonal channel resources, one for receiving and one for transmitting, i.e., the RS receives a signal on a first hop, applies signal processing to this signal and retransmits it on a second hop. In this paper, bi-directional communication with equal load from $S_1$ to $S_2$ and from $S_2$ to $S_1$ is assumed. For a one-way relaying approach, two resources are required for the transmission from $S_1$ to $S_2$ via the RS and two resources are required for the transmission from $S_2$ to $S_1$ via the RS leading to a requirement of overall four resources, i.e., compared to a bi-directional communication between $S_1$ and $S_2$ without two-hop relaying, the number of required resources is doubled. In the following, two different schemes are discussed which use multiple antennas at the RS in order to compensate for the increase in required resources in two-hop relaying.

For the first scheme, spatial diversity [4] is exploited at the RS in a one-way relaying approach. Maximum ratio combining (MRC) is a well-known approach for combating fading of the wireless channel [5]. Originally, signals which are received via multiple diversity branches are combined that way that the signal-to-noise ratio (SNR) at the receiver is maximized. In this case, channel state information (CSI) is required at the receiver side in order to determine proper weights for the multiple branches. If CSI is available at the transmitter side, MRC can also be applied to the transmit signal [6]. Assuming multiple antennas and CSI availability at the RS [7], one may apply both, receive and transmit MRC, since each antenna at the RS represents a diversity branch for reception as well as for transmission. In the following, the combination of receive and transmit MRC at the RS is termed MRC one-way relaying with a transceive filter matrix. MRC one-way relaying still requires four orthogonal resources for bi-directional communication, two resources for the transmission from $S_1$ to $S_2$ and two resources for the transmission from $S_2$ to $S_1$. However, it provides diversity gain which can compensate the increase in required resources by allowing higher transmission rates.

The second scheme has been introduced recently as multiple input multiple output (MIMO) two-way relaying [8]. In MIMO two-way relaying, $S_1$ and $S_2$ transmit simultaneously on the first channel resource to the RS. Like in MRC one-way relaying, a transceive filter matrix is applied at the multiple-antenna RS if CSI is available. The design of the transceive filter is divided into three steps. Firstly, the receive filter matrix separates the signals from $S_1$ and $S_2$. Secondly, the RS mapping matrix is introduced which ensures that each node is provided with its desired signal after retransmission from the RS. Thirdly, the transmit filter matrix is applied at the RS, which separates the signals designated to $S_1$ and $S_2$. The filtered signal is retransmitted on the second channel resource and received by $S_1$ and $S_2$. In this paper, MIMO two-way relaying is investigated for two linear transceive filters which fulfill the zero forcing (ZF) and the minimum mean square error (MMSE) constraint, respectively [9][10]. In MIMO two-way relaying, spatial diversity is not exploited. However, only two orthogonal channel resources are required for bi-directional communication, i.e., the multiple antennas at the RS are used to reduce the number of required channel resources.

Both schemes exploit that the RS is a receiver as well as a transmitter in the two-hop relaying case and both schemes require CSI at the RS. Like in a time division duplex (TDD) system, CSI for receive and transmit processing can be obtained by directly estimating the channel from $S_1$ to the RS and the channel from $S_2$ to the RS, respectively, and by exploiting channel reciprocity. Furthermore, both schemes substitute receive processing at $S_1$ and $S_2$ which makes knowledge of CSI unnessec-
Figure 1: Relaying scenario for $M^{(1)} = 1$ antenna at $S1$, $M^{(2)} = 1$ antenna at $S2$, and $M_{RS} = 2$ antennas at the RS.

In this paper, the approach of exploiting diversity in MRC one-way relaying is compared to the approach of reducing the number of required resources in MIMO two-way relaying. The performance of both schemes is investigated by means of the ergodic and the outage sum rate. The sum rate is defined as the sum of the mutual information values for bi-directional communication normalized by the number of required channel resources. The average performance of the schemes is described by the ergodic sum rate. The outage performance of the schemes can be analyzed by the outage sum rate which gives a measure for the diversity gain.

The paper is organized as follows: In Section II, a common system model for MRC one-way relaying and MIMO two-way relaying is introduced. In Section III, the sum rate for both schemes is derived. A comparison of MRC one-way relaying and MIMO two-way relaying with a linear ZF and MMSE transceive filter is given by means of simulations in Section IV. Finally, Section V concludes this work.

II. SYSTEM MODEL

In the following, a common system model for MRC one-way relaying and MIMO two-way relaying is derived. Concerning the system model, both relaying schemes only differ in the overall channel matrix and the overall transceive filter matrix which will be given in separate sections.

It is assumed that two nodes $S1$ and $S2$ communicate with each other which cannot exchange information directly, but via an intermediate RS. $S1$ and $S2$ are equipped with $M^{(1)} = M^{(2)} = M$ antennas and the RS is equipped with $M_{RS} = 2M$ antennas. In Fig. 1, the described scenario is depicted for the case of $M = 1$ and $M_{RS} = 2$.

Data vector $x^{(1)} = \left[ x_1^{(1)}, \ldots, x_M^{(1)} \right]^T$ of data symbols $x_n^{(1)}$, $n = 1, \ldots, M$, shall be transmitted from $S1$ to $S2$, and data vector $x^{(2)} = \left[ x_1^{(2)}, \ldots, x_M^{(2)} \right]^T$ of data symbols $x_n^{(2)}$, $n = 1, \ldots, M$, shall be transmitted from $S2$ to $S1$, where $[\cdot]^T$ denotes the transpose. The overall data vector is defined by $\hat{x} = \left[ x^{(1)\dagger} \cdot x^{(2)\dagger} \right]^T$. Since spatial filtering shall only be applied at the RS, only scalar transmit filters $Q^{(1)} = q^{(1)}I_M$ and $Q^{(2)} = q^{(2)}I_M$ are applied at $S1$ and $S2$, where $I_M$ is an identity matrix of size $M$. These transmit filters are required in order to fulfill the transmit energy constraints. Assuming that $E^{(1)}$ and $E^{(2)}$ are the maximum transmit energies of nodes $S1$ and $S2$, the transmit energy constraints are given by

$$E \left\{ \| q^{(k)}x^{(k)} \|_2^2 \right\} \leq E^{(k)}, \quad k = 1, 2, \quad (1)$$

where $E \{ \cdot \}$ and $\| \cdot \|_2$ denote the expectation and the Euclidian norm, respectively. The overall transmit filter is given by the block diagonal matrix

$$Q = \begin{bmatrix} Q^{(1)} & 0 \\ 0 & Q^{(2)} \end{bmatrix}$$

(2)

where $0_M$ is a null matrix with $M$ rows and $M$ columns. For simplicity, but without loss of generality, the wireless channel is assumed to be flat fading, i.e., all following considerations are applicable to multi-carrier systems. Hence, the channel from $S_k$, $k = 1, 2$, to the RS may be described by the channel matrix

$$H^{(k)} = \begin{bmatrix} h_{1,1}^{(k)} & \ldots & h_{1,M}^{(k)} \\ \vdots & \ddots & \vdots \\ h_{M,1}^{(k)} & \ldots & h_{M,M}^{(k)} \end{bmatrix},$$

(3)

where $h_{m,n}^{(k)}$, $m = 1, \ldots, M_{RS}$ and $n = 1, \ldots, M$, are complex fading coefficients. Note that the channel matrix from the RS to node $S_k$ is the transpose $H^{(k)\dagger}$ of channel matrix $H^{(k)}$ assuming that the channel is constant during one transmission cycle from $S1$ to $S2$ and from $S2$ to $S1$. The overall channel matrix $H$ depends on the underlying relaying protocol at the RS. At the RS, a linear transceive filter $G$ is applied to the received signal. Before retransmission, the filtered transmit vector $x_{RS}$ has to fulfill the transmit energy constraint at the RS

$$E \left\{ \| x_{RS} \|_2^2 \right\} \leq E_{RS},$$

(4)

where $E_{RS}$ is the maximum transmit energy at the RS. Since spatial filtering shall only be applied at the RS, only scalar receive filters $P^{(1)} = p^{(1)}I_M$ and $P^{(2)} = p^{(2)}I_M$ are assumed at $S1$ and $S2$. The overall receive filter $P$ is given by a block diagonal matrix

$$P = \begin{bmatrix} P^{(1)} & 0_M \\ 0_M & P^{(2)} \end{bmatrix}.$$  

(5)

In the following, the estimate for data vector $x_2$ at $S1$ is termed $\hat{x}_1$ and the estimate for data vector $x^{(1)}$ at $S2$ is termed $\hat{x}^{(2)}$. The overall estimated data vector $\hat{x} = \left[ \hat{x}^{(1)\dagger} \cdot \hat{x}^{(2)\dagger} \right]^T$ after the scalar receive filter is given by

$$\hat{x} = P \left( H^T GH \hat{x} + H^T G_{n_{RS}} + n_R \right),$$

(6)

where it is assumed that $n_{RS}$ and $n_R$ are additive noise vectors at the RS and at $S1$ and $S2$, respectively, with $n_{RS} = n_{R}^{(1)}T n_{R}^{(2)}T$ and $n_{R}^{(1)}$ and $n_{R}^{(2)}$ being the noise vectors at $S1$ and $S2$, respectively.
A. Maximum Ratio Combining at the RS

For MRC one-way relaying, the bi-directional communication between S1 and S2 requires four orthogonal time slots. During the first time slot, S1 transmits $x^{(1)}$ to the RS. Firstly, receive MRC [5] is applied to the receive vector at the RS. The receive filter which is matched to channel $H^{(1)}$ is given by

$$G_R^{(1)} = \frac{H^{(1)\dagger}}{\|H^{(1)\dagger}\|_2^2}$$

(7)

where $\|H^{(1)\dagger}\|_2^2$ normalizes the receive filter. Secondly, transmit MRC [6] is applied which is described by the transmit filter

$$G_T^{(2)} = c_T^{(2)}H^{(2)\dagger}$$

(8)

matched to channel $H^{(2)\dagger}$ where $c_T^{(2)}$ is positive real-valued in order to meet the transmit energy constraint at the RS from Eq. (4). The overall transceive filter at the RS for the transmission from S1 to S2 results in

$$G^{(1)} = G_T^{(2)}G_R^{(1)}$$

(9)

During the second time slot, the RS retransmits the filtered vector to S2 leading to the estimate

$$\hat{x}^{(2)} = P^{(2)}\left(H^{(2)\dagger}G^{(1)}H^{(1)\dagger}Q^{(1)}x^{(1)} + H^{(2)\dagger}G^{(1)}n_{RS} + n_{R}^{(2)}\right).$$

(10)

During the third time slot, S2 transmits $x^{(2)}$ to the RS. The receive vector at the RS is filtered by the overall transceive filter which is given by

$$G^{(2)} = G_T^{(1)}G_R^{(2)}$$

(11)

where the derivation of the receive filter $G_R^{(2)}$ and the transmit filter $G_T^{(1)}$ is the same as for $G_R^{(1)}$ and $G_T^{(2)}$ in Eqs. (7) and (8), respectively. During the fourth time slot, the RS retransmits the filtered vector to S1 leading to the estimate

$$\hat{x}^{(1)} = P^{(1)}\left(H^{(1)\dagger}G^{(2)}H^{(2)\dagger}Q^{(2)}x^{(2)} + H^{(1)\dagger}G^{(2)}n_{RS} + n_{R}^{(1)}\right).$$

(12)

Finally, with the overall channel matrix

$$H = \left[\begin{array}{cc} H^{(1)} & 0 \\ 0 & H^{(2)} \end{array}\right]$$

(13)

and with the overall transceive filter matrix

$$G = \left[\begin{array}{cc} 0 & G^{(1)} \\ G^{(2)} & 0 \end{array}\right]$$

(14)

equations (10) and (12) can be jointly described by Eq. (6).

B. MIMO Two-Way Relaying

For MIMO two-way relaying from [11], the bi-directional communication between S1 and S2 requires two orthogonal time slots. During the first time slot, S1 and S2 transmit $x^{(1)}$ and $x^{(2)}$ simultaneously to the RS. Firstly, a linear receive filter $G_R$ is applied at the RS which separates the transmitted vectors $x^{(1)}$ and $x^{(2)}$. For example, the receive filter may be designed according to the linear ZF or MMSE criterion given in [8]. Secondly, an RS mapping matrix

$$G_{\Pi} = \begin{bmatrix} 0 & I \\ I & \emptyset \end{bmatrix}$$

(15)

is applied which ensures that the RS transmits the estimate for $x^{(2)}$ in the direction of S1 and the estimate for $x^{(1)}$ in the direction of S2. Thirdly, a linear transmit filter $G_T$ is applied which separates the vectors designated for S1 and S2, respectively. For example, the transmit filter may be designed according to the linear ZF or MMSE criterion given in [8]. During the second time slot, the RS retransmits the filtered vector to S1 and S2 simultaneously. With the overall transceive filter

$$G = G_TG_{\Pi}G_R$$

(16)

and the overall channel matrix

$$H = \left[\begin{array}{cc} H^{(1)} & H^{(2)} \end{array}\right].$$

(17)

the overall estimated data vector $\hat{x}$ is described by Eq. (6).

III. SUM RATE OF ONE-WAY AND TWO-WAY RELAYING

In the following, the sum rate of a system is defined as the sum of the mutual information values for bi-directional communication normalized by the number of required channel resources. For purposes of further investigations concerning the sum rate of the MRC one-way relaying approach and the MIMO two-way relaying approach, Eq. (6) may be rewritten as

$$\hat{x} = Ax + [D \quad P]n$$

$$\hat{x} = \begin{bmatrix} A^{(1)} \\ A^{(2)} \end{bmatrix}x + \begin{bmatrix} B^{(1)} \\ B^{(2)} \end{bmatrix}n$$

(18)

with

$$A = PH^TGHQ$$

$$D = PH^T$$

$$n = \left[n_{RS}^T \quad n_{R}^T\right]^T$$

(19)

(20)

(21)

leading to the two estimates at nodes S1 and S2

$$\hat{x}^{(k)} = A^{(k)}x + B^{(k)}n \quad \text{for} \quad k = 1, 2$$

(22)

with $A^{(k)}$ of dimension $M \times 2M$ and $B^{(k)}$ of dimension $M \times 4M$.

From [12], it can be shown that the mutual information between receive node $k$ and the corresponding transmit node is given by

$$C^{(k)} = \log_2 \left(\det \left[I_M + \frac{A^{(k)}R_xA^{(k)H}}{B^{(k)}R_nB^{(k)H}}\right]\right) \quad \text{for} \quad k = 1, 2$$

(23)

where $[\cdot]^H$ denotes the conjugate complex transpose, $\log_2 (\cdot)$ denotes the logarithm to the basis 2, $\det [\cdot]$ denotes the determinant, and $R_x$ and $R_n$ are the overall transmit vector and
For MRC one-way relaying, the transmissions from $S_1$ to $S_2$ and from $S_2$ to $S_1$ are separated by four orthogonal channel resources of equal size. The overall sum rate of MRC one-way relaying is defined as

$$C_{OW} = \frac{1}{4} \left( C_{OW}^{(1)} + C_{OW}^{(2)} \right)$$

where $C_{OW}^{(1)}$ is the mutual information for the transmission from $S_2$ to $S_1$, $C_{OW}^{(2)}$ is the mutual information for the transmission from $S_1$ to $S_2$, and the pre-log factor $1/4$ is introduced in order to indicate the number of required channel resources for the bi-directional communication.

For MIMO two-way relaying, the communication takes place in two directions simultaneously by using two orthogonal channel resources. Therefore, the sum rate of MIMO two-way relaying is defined as

$$C_{TW} = \frac{1}{2} \left( C_{TW}^{(1)} + C_{TW}^{(2)} \right)$$

where $C_{TW}^{(1)}$ is the mutual information for the transmission from $S_2$ to $S_1$, $C_{TW}^{(2)}$ is the mutual information for the transmission from $S_1$ to $S_2$, and the pre-log factor $1/2$ is introduced in order to indicate the number of required channel resources for the bi-directional communication.

### IV. Simulation Results

In this section, the sum rate of MRC one-way relaying is compared to the sum rate of MIMO two-way relaying with a linear ZF and MMSE transceive filter at the RS by means of simulations. The transmit and receive filters $Q$ and $P$ at $S_1$ and $S_2$ and the transceive filter $G$ at the RS from Section II are chosen according to the considered relaying scheme. In all cases, it is assumed that nodes $S_1$ and $S_2$ are each equipped with $M = 1$ antenna and the RS is equipped with $M_{RS} = 2$ antennas. The channel coefficients are spatially white and Rayleigh distributed with zero mean and variance $1$. The noise vectors are complex zero mean Gaussian with variance $\sigma^2_{RS}$ at the RS, variance $\sigma^2_1$ at $S_1$, and variance $\sigma^2_2$ at $S_2$, respectively. The presented results are achieved from Monte Carlo simulations with statistically independent channel fading realisations where $\rho^{(1)} = E_{RS}/\sigma^2_1 = E^{(1)}/\sigma^2_{RS}$ denotes the average SNR between $S_1$ and the RS and $\rho^{(2)} = E_{RS}/\sigma^2_2 = E^{(2)}/\sigma^2_{RS}$ denotes the average SNR between $S_2$ and the RS.

Fig. 2 gives the ergodic sum rate for the three transceive filters depending on $\rho^{(2)}$ with $\rho^{(1)} = 10$dB, 20dB. For increasing $\rho^{(2)}$, the sum rate converges to a constant maximum which depends on $\rho^{(1)}$. For small $\rho^{(1)}$, the sum rate converges faster with increasing $\rho^{(2)}$ and the maximum sum rate is lower than for high $\rho^{(1)}$. Obviously, the linear ZF and MMSE transceive filters in the MIMO two-way relaying approach outperform the MRC one-way relaying approach in terms of ergodic sum rate. This comes from the fact that in MIMO two-way relaying, both directions of communication are processed simultaneously while in MRC one-way relaying only one direction of communication is processed. For low $\rho^{(2)}$, the relative gain of MIMO two-way relaying with a linear MMSE transceive filter compared to MRC one-way relaying is higher than $2$. In the saturation region, the relative gain is slightly decreased. For low $\rho^{(2)}$, the linear ZF transceive filter has a worse performance than the linear MMSE. But for increasing $\rho^{(2)}$, the sum rate of the linear ZF filter converges to the MMSE solution since the receiver noise can be neglected.

Fig. 3 gives the $10\%$ outage sum rate, i.e., the sum rate which is exceeded in $90\%$ of the cases, depending on $\rho^{(2)}$ with $\rho^{(1)}$ as a parameter. The linear MMSE transceive filter for MIMO two-way relaying still outperforms the MRC one-way relaying approach. However, especially for increasing $\rho^{(2)}$ the relative gain of MIMO two-way relaying compared to MRC one-way relaying is lower for the $10\%$ outage sum rate than for the ergodic sum rate. MRC one-way relaying exploits spatial diversity which directly improves the outage performance and the gain due to diversity increases with increasing SNR [4]. In contrast to MRC one-way relaying, MIMO two-way relaying does not exploit spatial diversity since the multiple antennas are used for spatial separation of the signals from $S_1$ and $S_2$ by beamforming. The outage sum rate of the linear ZF transceive filter in MIMO two-way relaying is even below the outage sum rate of the MRC one-way relaying approach for $\rho^{(1)} = 10$dB. For $\rho^{(1)} = 20$dB, it is below the MRC one-way relaying approach if $\rho^{(2)} \leq 26$dB. This comes from the fact that the linear ZF transceive filter simply reverses the current fading channel without considering the current SNR, i.e., especially for channels in deep fade the linear ZF filter has a very bad performance. These deep fades of the channel coefficients have a higher impact for low SNR which explains that the linear ZF transceive filter performs worse than MRC one-way relaying for low SNR and performs as well as the MMSE transceive filter for high SNR.

The bad outage performance of the ZF transceive filter be-

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This paper considers how multiple antennas and availability of CSI at the RS can be exploited in two-hop relaying. MRC one-way relaying and MIMO two-way relaying are discussed as possible approaches. Both approaches substitute receive processing at the communicating nodes $S1$ and $S2$ which makes knowledge of CSI unnecessary at $S1$ and $S2$ and promises to reduce the CSI signaling effort in relay networks. Considering the average performance, the MIMO two-way relaying approach always provides better results than the MRC one-way relaying approach. However, it is shown that the outage performance in MIMO two-way relaying significantly depends on the choice of the linear filter at the RS. A linear filter fulfilling the ZF constraint provides a worse outage performance than the MRC one-way relaying approach. In contrast, a linear filter fulfilling the MMSE constraint achieves a relative gain higher than 2 compared to MRC one-way relaying for low to medium SNR values.

V. CONCLUSION

This paper considers how multiple antennas and availability of CSI at the RS can be exploited in two-hop relaying. MRC one-way relaying and MIMO two-way relaying are discussed as possible approaches. Both approaches substitute receive processing at the communicating nodes $S1$ and $S2$ which makes knowledge of CSI unnecessary at $S1$ and $S2$ and promises to reduce the CSI signaling effort in relay networks. Considering the average performance, the MIMO two-way relaying approach always provides better results than the MRC one-way relaying approach. However, it is shown that the outage performance in MIMO two-way relaying significantly depends on the choice of the linear filter at the RS. A linear filter fulfilling the ZF constraint provides a worse outage performance than the MRC one-way relaying approach. In contrast, a linear filter fulfilling the MMSE constraint achieves a relative gain higher than 2 compared to MRC one-way relaying for low to medium SNR values.

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